0505N269

Candidate's Seat No :________

MSc Sem.-2 Examination

410

Mathematics

May-2025

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Find the general integral of the equation **(07)**

$$(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$$

(b) Prove that if $\vec{X} \cdot curl\vec{X} = 0$ where $\vec{X} = (P, Q, R)$ and μ is an arbitrary differentiable function of x, y and z then $\mu\vec{X} \cdot curl(\mu\vec{X}) = 0$. **(07)**

OR

Q.1 **(a)** Check compatibility of given equations and find their solution **(07)**

$$xp - yq = x\,, x^2p + q = xz$$

(b) Find the complete integral of the equation **(07)**

$$q = (z + px)^2$$

Q.2 **(a)** Find a complete integral of the equation $p^2x + q^2y = z$ by Jacobi's method. **(07)**

(b) Find the integral surface of the equation **(07)**

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$

which passes through the line $x = 1, y = 0$.

OR

Q.2 **(a)** Find a complete integral of the equation **(07)**

$$p + q^2 = 0$$

and the integral surface passing through the line $x_0(s) = 0, y_0(s) = s, z_0(s) = 3s$.

(b) Find the characteristics of the equation **(07)**

$$pq = 1$$

which containing the initial line $x_0 = 2s, y_0 = 2s, z_0 = 5s$.

Q.3 **(a)** Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form. **(07)**

(b) State and solve the heat conduction problem for a finite rod of length l with initial temperature distribution in the rod at time $t = 0$ given by $f(x)$. Use the method of separation of variables. **(07)**

OR

Q.3 **(a)** State and solve the problem of the wave equation in case of an infinite vibrating string with initial displacement distribution $f(x)$ and initial velocity distribution $g(x)$. **(07)**

(b) A bar of 10 cm. length with insulated sides A and B are kept at 20°C and 40°C respectively until steady state condition prevail. The temperature at A is then suddenly varied to 50°C and the same instant at B, lowered at 10°C. Find the subsequent temperature at any point of the bar at any time. **(07)**

(P.T.O)

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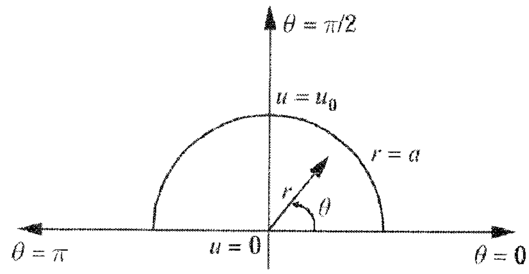
Q.4 **(a)** State and solve the Dirichlet problem for a rectangle. **(07)**

(b) State and solve Neumann problem for a circle. **(07)**

OR

Q.4 **(a)** State and solve Dirichlet problem for the upper half plane. **(07)**

(b) Find the steady state temperature distribution in a semi-circular plate of radius a, insulated on both the faces with its curved boundary kept at a constant temperature U_0 and its bounding diameter kept at zero temperature as described in Fig. **(07)**



Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Eliminate the arbitrary function from the following equation and hence, obtain the corresponding partial differential equation

$$z = f\left(\frac{xy}{z}\right)$$

(2) Find the complete integral of the equation $z = px + qy + p^2 + q^2$

(3) Define general integral of the first order PDE $f(x, y, z, p, q) = 0$.

(4) Write down necessary and sufficient conditions under which the equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ are compatible.

(5) Write down the strip condition.

(6) For what values of x and y are the equation $u_{xx} + 2xu_{xy} + (1 - y^2)u_{yy} = 0$ hyperbolic?

(7) Find the characteristic of $(\sin^2 x)r + (2\cos x)s - t = 0$

(8) Define: Mixed Boundary Condition

(9) State Harnack's theorem.

(10) What is the necessary condition for the existence of the solution U of the problem $\nabla^2 U = 0$ in a bounded domain D, and $\frac{\partial U}{\partial n} = f(x)$ on the boundary B, where $\frac{\partial}{\partial n}$ is the directional derivative along the outward normal?

(11) Define: Neumann boundary value problem

(12) Define: Family of equipotential surfaces

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