0505N287

Candidate's Seat No :________

MSc Sem.-2 Examination

410

AMS

May-2025

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Solve $\frac{dy}{dx} = y + e^x$, $y(0) = 1$ for $x = 0.2$ by modified Euler Method **(07)**

(b) Find $y(0.1)$ and $y(0.2)$, given that $y' = y - \frac{2x}{y}$, $y(0) = 1$ using Euler predictor-corrector method. **(07)**

OR

(a) Using the fourth order Runge-Kutta formulas, find $y(0.2)$ given that $y' = x + y, y(0) = 1$ **(07)**

(b) Evaluate $I = \int_0^6 \frac{1}{1+x} dx$ using Simpson's 1/3 rule. **(07)**

Q.2 **(a)** Using the Laplace transforms, find the solution of the initial value problem **(07)**

$y'' + 25y = 10\cos 5t$, $y(0) = 2$ *and* $y'(0) = 0$

(b) Using the convolution theorem, find **(07)**

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}, a \neq b$$

OR

(a) Find the inverse Laplace transform of **(07)**

$$\frac{s+4}{s(s-1)(s^2+4)}$$

(b) State convolution theorem and by using the theorem obtain **(07)**

$$L^{-1}\frac{1}{s(s^2+a^2)}$$

Q.3 **(a)** Determine if $f(x) = x^3 + \sin(x)$ is even, odd, or neither. Write the Fourier sine series for $f(x) = x^3$ on $[0, \pi]$. **(07)**

(b) Find the Fourier sine transform of $f(x) = \begin{cases} x \,;\, 0 < x \leq 1 \\ 2 - x \,;\, 1 < x < 2 \\ 0 \,;\, x > 2 \end{cases}$. **(07)**

OR

(a) I. Determine the half-range cosine expansion for $f(x) = e^x$ on $[0, L]$. **(07)**

II. Compute the Fourier series for $f(x) = x$ with period 2L. Find the coefficient of a_n.

(b) Find the Fourier integral representation of the function: **(07)**

$$f(x)=\begin{cases}2, & |x|<2\\ 0, & |x|>2\end{cases}$$

Q.4 **(a)** Find the analytic function $w = u + iv$ whose imaginary part is given by $v = e^x(x\sin y + y\cos y)$. **(07)**

(b) Find the image of the line $x - y = 1$ under the transformation $w = \frac{1}{z}$ **(07)**

OR

(a) State Cauchy's Integral Theorem. **(07)**
Evaluate $\oint_C (z^2 + 3)dz$, where C is any closed contour. Justify your answer.

(b) State Cauchy's Residue Theorem. **(07)**
Evaluate $\oint_C \frac{z^2-4z+4}{z+i}dz$, where C is $|z| = 2$

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Define Trapezoidal rule.

(2) Using Euler's method.
find $y(0.1)$ given $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$ with $h = 0.1$.

(3) State the basic difference between the limit of a function of a real variable and that of a complex variable.

(4) Find the residue at $z = 0$ of $f(z) = z\cos\frac{1}{z}$.

(5) Define: Essential Singularity with suitable example.

(6) Which term appears in the Fourier series of an even function?

A) Only sine terms
B) Only cosine terms
C) Both sine and cosine terms
D) Neither sine nor cosine terms

(7) The half-range expansion of a function is useful for:

A) Extending the function to an even or odd function
B) Reducing the period of the function
C) Making the function discontinuous
D) Eliminating Fourier coefficients

(8) Find the Fourier coefficient a_0 for the function $f(x) = x$ defined on $(-\pi, \pi)$.

A) 0
B) π
C) $-\pi$

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D) $\frac{\pi^2}{2}$

(9) If $L[f(t)] = F(s)$, then $L\,[\frac{1}{t} f(t)]$ is:

A) $\int_s^{\infty} F(s)$

B) $\int_s^{\infty} s\,F(s)$

C) $\frac{F(s)}{s}$

D) None of the above

(10) Laplace transform of $f(t) = e^{at}$ is

A) $\frac{1}{s}$ *where* $s > 0$

B) $\frac{1}{s-a}$ *where* $s > a$

C) $\frac{1}{s+a}$

D) None of the above

(11) Choose the correct option

A) $L(1) = \frac{1}{s}$

B) $L(t) = \frac{1}{s}$

C) Both A and B

D) None of the above

(12) Which of the following numerical integration methods uses parabolic segments to approximate the area under a curve?

A) Trapezoidal Rule

B) Midpoint Rule

C) Simpson's 1/3 Rule

D) Monte Carlo Method

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