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0505N254

Candidate's Seat No :_____

B.C.A. Sem.-2 Examination

CC-111

Discrete Mathematics (New)

May-2025

Time : 2-30 Hours] **[Max. Marks : 70**

1 A Let a set G = $\{(a, b) / a, b \in Z\}$ with operation * defined on it as follows: 7
For any $\alpha = (a_1, b_1)$, $\beta = (a_2, b_2) \in G$, $\alpha * \beta = (a_1+a_2, b_1+b_2)$. Show that (G, *) is an abelian group.

B Show that a set $(G, +_5)$ is a group, where G = {0, 1, 2, 3, 4} and '$+_5$' denotes the operation addition reduced modulo 5. Also find the order of each element of it. 7

OR

A Show that a set {1, -1, i, -i} with operation multiplication is a group. (where i is the imaginary unit in complex number system.) 7

B Prove that the identity element of a Group is unique. 7

2 A Find covers of each of the elements of a Poset $\langle P(A), \subseteq \rangle$, where A= {1, 2, 3} and draw Hass diagram for this Poset. 7

B Let a set X = {1, 2, 3, 4} and relations on it as follows: 7
R_1 = {(1, 1), (1, 3), (2, 3), (2, 4)}
R_2 = {(1, 1), (2, 2), (2, 3), (3, 2), (3, 3), (3, 4), (4, 3), (4, 4)}
Answer the following questions.
a. Give the Range of a relation R_1.
b. Write $R_3 = R_1 \cup R_2$. Is the relation R_3 an anti-symmetric?
c. Give Relation Matrix for the relation R_2.

OR

A Let Z be the set of integers and define a relation R = {(x, y) | x – y is divisible by 3} on Z. Show that given relation R is an equivalence relation. Also show that the set of all equivalence classes is a partition of Z. 7

B Define POSET. Show that a set S_{60} (S_m is a set of divisors of m) is a POSET with respect to a relation D (divides) on it. 7

3 A Show that $\langle S_{60}$, glb, lub$\rangle$ with partial ordering D (divides) and g.l.b.(a, b) = G.C.D.(a, b) , l.u.b.(a, b) = L.C.M.(a, b), is a lattice. 7

B Let a Poset $\langle P(A), \subseteq \rangle$, where A = {1, 2, 3}. First show that $\langle P(A), \subseteq \rangle$ is a lattice and then show that it is an Boolean Algebra with respect to GLB(X, Y) = X∪Y, LUB(X, Y) = X∩Y and X′ = complement of a set X. 7

OR

A Show that $(a * b)' = a' \oplus b'$ hold in a complemented, distributive lattice. 7

B Which of the two lattices $\langle S_n, D \rangle$ for n = 30 and n = 45 are complemented? If They are complemented then find complements of its elements. 7

4 A Draw graphs from the following adjacency matrices. Are these graphs isomorphic? Justify your answer. 7

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \textit{and} \quad \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

P.T.O.

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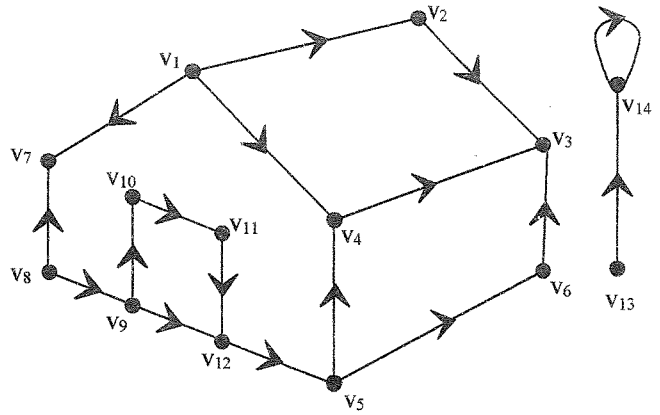
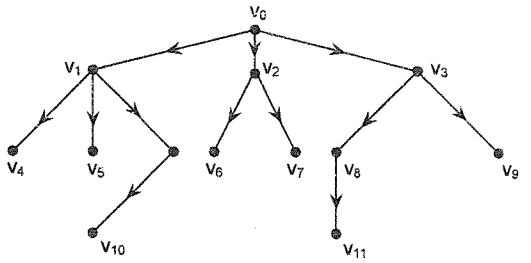
B Give the adjacency matrix for the following tree. 7
$(V_1 (V_2 (V_6)(V_7)) (V_3 (V_8)) (V_4) (V_5 (V_9)(V_{10})(V_{11})))$

OR

A Find the reachable set for the following sets of vertices from the given digraph. 7
(a) $\{v_1\}$ **(b)** $\{v_1, v_8\}$ **(c)** $\{v_1, v_8, v_{13}\}$



B Give other representations of the given tree. Also give the binary tree representation of it. 7



5 Attempt any seven out of twelve. 14

1. A group has unique identity element. (True / False)
2. The set Z of integers is a group with respect to operation _____.
   (a) Addition **(b)** Multiplication **(c)** $a * b = (a + b)/2$ **(d)** None of these
3. In the additive group of integers the order of every element other than 0 is _____ .
   (a) Zero **(b)** n **(c)** infinite **(d)** None of these
4. Every subgroup of an abelian group is abelian. (True / False)
5. A Relation is reflexive then it is not irreflexive. (True / False)
6. A covering of a set is always a partition of that set. (True / False)
7. A relation is symmetric if its matrix is _____ .
   (a) Symmetric **(b)** Anti-symmetric **(c)** Square **(d)** None of these
8. Partial ordering relation is an equivalence relation. (True / False)
9. Every Poset is a lattice. (True / False)
10. Every Boolean Algebra is a lattice. (True / False)
11. $\langle S_6, D\rangle$ is a sublattice of a lattice $\langle S_{12}, D\rangle$. (True / False)
12. Every subset of a lattice is a sublattice. (True / False)