0305N222

Candidate's Seat No :________

MSc Sem.-2 Examination

409

AMS

Time : 2-30 Hours] **May-2025** **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Explain Harrod Model (Economic Model): Objective, assumptions and conclusion with suitable equations. Also, define Warranted Growth rate, Actual Growth rate and Natural Growth rate. **(07)**

(b) Let a town be affected with viral fever. Each day 10% of those who have the viral fever in the town recover from it, while another 500 people are affected with the viral fever. If there are currently 2,000 cases of viral fever,
Formulate a discrete mathematical model and find out how many cases will be there, two weeks from now. **(07)**

OR

(a) Explain Linear Prey-Predator Model: Assumptions, equilibrium point, stability analysis with suitable example. **(07)**

(b) The dynamical system that models the amount of alcohol in a person's body is given by $A_{n+1} = A_n - \frac{9A_n}{4.2+A_n} + d$ where A_n is the number of grams of alcohol in the body at the beginning of hour n and d is the constant amount consumed per hour. Find the equilibrium value, given that this person consumes 7 grams of alcohol per hour. Is the system stable? **(07)**

Q.2 **(a)** Consider a model of species competing for food and space. The governing equation is given by **(07)**

$$\frac{dx}{dt} = \alpha x - \beta y$$
$$\frac{dy}{dt} = \gamma y - \delta x$$

where x and y are two competing species, $\alpha, \beta, \gamma, \delta$ are positive constants.
Show that $\frac{d^2x}{dt^2} - (\alpha + \gamma)\frac{dx}{dt} + (\alpha\gamma - \beta\delta)x = 0$ and solve for x.

(b) Explain Bifurcations and state its classifications. Discuss Transcritical Bifurcation in brief. **(07)**

OR

(a) Explain Richardson Arms Race Model: Assumptions, Stability analysis with equilibrium points and discuss all the different cases. **(07)**

(b) A mathematical model for epidemics consisting of susceptible(S), infected(I) and removals(R) is given by **(07)**

$$\frac{dS}{dt} = -\beta S^2 I, \quad \frac{dI}{dt} = \beta S^2 I - \gamma I, \quad \frac{dR}{dt} = \gamma I$$

where β and γ are positive constants.

i. Find the threshold density of susceptible.

ii. Show that $\frac{dR}{dt} = \gamma\left\{n - R - \frac{RS_0\beta}{\gamma}\right\}$

(P.T.O)

Q.3 **(a)** Explain the Epidemic model which concludes that as time increases, all the susceptible persons will be infected. **(07)**

(b) Define Traffic density function. **(07)**
Find the traffic density $\rho(x,t)$, satisfying $\frac{\partial \rho}{\partial t} + (xsint)\frac{\partial \rho}{\partial x} = 0$ with initial condition $\rho_0(x) = 1 + \frac{1}{1+x^2}$

OR

(a) The favorite food of the tiger shark is the sea turtle. A two-species pre-predator model is given by **(07)**

$$\frac{dP}{dt} = P(a - bP - cS)$$
$$\frac{dS}{dt} = S(-k + \mu P),$$

where P is the sea turtle, S is the shark and $a, b, c, k, \mu > 0$.

i. Let $b = 0$ and the value of k is increased. Ecologically, what is the interpretation of increasing k and what is its effect on the non-zero equilibrium populations of sea turtle and sharks?

ii. Obtain all the equilibrium solutions for $b = 0$.

(b) Derive the spatial Lake pollution model with assumptions and approach $\frac{\partial C}{\partial t} + \frac{F}{A}\frac{\partial C}{\partial x} = 0$ with boundary conditions $C(a,t) = \frac{g(t)}{F}$, $C(x,0) = P(x)$ using formulation of PDE. **(07)**

Q.4 **(a)** Define Delay Differential Equations (DDE)? **(07)**
Solve Delay Differential Equation:

$$\frac{dx}{dt} = -x(t-\tau), t > 0$$

Initial history: $x(t) = 1, -\tau \le t \le 0.$

(b) What is Traffic Flow? State objectives of Traffic Flow Modeling. Its general assumptions, classifications and Traffic density. **(07)**

OR

(a) State different types of Delay Differential Equations (DDE). **(07)**
Solve the following DDE using the method of steps:

$$\dot{x}(t) = -kx(t-x),\ x(t) = a,\ -r \le t \le 0,$$

where k, a and r are constants, $r > 0$.

(b) Explain in brief: The Theory of Car-Following **(07)**

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Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Obtain the difference equation by eliminating the arbitrary constants from $u_n = A(3)^n + B(-2)^n$

(2) What is Mathematical Modelling? State the (basic) types of models.

(3) Choose the correct Bifurcation whose normal form is given by $\dot{x} = \mu x - x^3$.

A. Saddle-Node Bifurcation
B. Transcritical Bifurcation
C. Hoff Bifurcation
D. Pitchfork Bifurcation

(4) Diseases that are always present in a community, usually at a low, more or less constant, frequency are classified as having an ____ pattern.

A. Epidemic
B. Endemic
C. Pandemic
D. Hyper pandemic

(5) An infection experiences deterministic extinction if the basic reproductive ratio R_0 is

A. Equal to 1
B. Less than 1
C. Greater than 1
D. Between -1 and 1

(6) Draw Bifurcation diagram of Supercritical pitchfork bifurcation.

(7) What is a sigmoid growth curve called?

A. Exponential growth curve
B. Declining growth curve
C. Logistic growth curve
D. Interacting curve

(8) State: Equation of conservation of mass or continuity equation.

(9) Write the spatial aspect of the Prey-Predator Model with diffusion term.

(10) State normal form of Saddle-Node Bifurcation.

(11) State Routh-Hurwitz criteria for Cubic Equation: $\mu^3 + a_1\mu^2 + a_2\mu + a_3 = 0$

(12) State the compartment of SIRS Model.

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