Seat No. : _______________

11C-128

May-2015

M.Sc., Sem.-II

408 : Statistics

(Distribution Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions carry equal marks.
(2) Scientific calculator can be used.

1. (a) Let $X_1, X_2, \ldots X_N$ are N identically independently distributed random variables and N is also a random variable independent of X_i's. If $S_N = Y = \sum_{i=1}^{N} X_i$, then show that (i) $E(S_N) = E(N)\,E(X)$ (ii) $V(S_N) = E(N)V(X) + V(N)\{E(X)\}^2$

OR

Let $X_1, X_2, \ldots X_n$ are independent discrete random variables and N is also a random variable independent of X_i's. Let $Y = \sum_{i=1}^{N} X_i$ and ϕ_i, i = 1, 2, 3 are the characteristic functions of random variables N, X and Y respectively. Express characteristic function ϕ_3 as a compound function of ϕ_1 and ϕ_2.

(b) Define Neyman type – A distribution. Obtain its probability Generating function. Hence derive its r^{th} factorial cumulant. Also describe the method of fitting of Neyman type-A distribution to the numerical data.

OR

Define Poisson –Binomial distribution. Obtain its probability generating function. Show that Poisson – Binomial distribution tends to Poisson – Poisson distribution. State necessary assumptions involved.

2. (a) Discuss the roll of non-central distributions in statistical inference with illustration. If $X \tilde{} N(\mu, 1)$ then, obtain probability density function of non-central Chi-square distribution using moment generating function.

OR

Define Poisson – Pascal distribution. Obtain recurrence relations for Probabilities and descending factorial moments for this distribution.

(b) Define non-central 'F' distribution with (n_1, n_2) degrees of Freedom. In usual notations, obtain probability density function of non-central 'F' distribution.

OR

Define non-central 't' statistic. In usual notations obtain probability density function of non-central 't' distribution.

3. (a) Define order statistics. Obtain the distribution of the r^{th} order statistics. Also obtain the Probability density function of the r^{th} order statistics.

OR

Let a random variable 'X' follows an Exponential distribution with mean θ, $\theta > 0$. If a random sample of size n is taken from this distribution, then find out the probability density function of $X_{(r)}$.

(b) Define the sample range. Obtain the distribution of sample range for infinite range population. State the distribution of sample range for finite range population.

OR

If a random sample of size 'n' is taken from the exponential distribution with mean 1/3,then find the probability that the sample range does not exceed 2.

4. (a) If $X_{(n)} = \max \{X_1, X_2 \ldots\ldots X_n\}$, then show that

$$E(X_{(n)}) = E(X_{(n-1)}) + \int_0^\infty F^{(n-1)}(x)\,(1 - F(x))\,dx.$$

OR

Define rank-order statistics with appropriate example. Give functional definition of rank-order statistics. In usual notations obtain the formula for the correlation coefficient between the rank-orders and variate values.

(b) Obtain the mean and variance of r^{th} order statistic for the uniform distribution U(0, 1).

OR

Explain the procedure of obtaining Confidence Interval for p^{th} Quantile of the distribution. If X(r) be the r^{th} order statistic of a random sample of size 7 taken from any continuous distribution with cumulative distribution function $F_x(X)$, then obtain $P(X_{(3)} <$ Population median $< X_{(5)})$.

5. Choose the correct answer.

(i) If $X_1, X_2, \ldots, X_m, X_{m+1} \ldots, X_{m+n}$ are independent normal variates with zero mean and standard deviation σ, then the distribution of $\sum_{i=1}^{m} X_i^2 / \sum_{i=m+1}^{m+n} X_i^2$ is

(a) F(m, n) (b) F(m, m + n)

(c) F'_λ (m, n) (d) None of these

(ii) If $X_1, X_2, \ldots, X_n$, are independent variates each distributed as $N(0, \sigma^2)$, then the probability density function of $w = X_1 / \left(\frac{1}{n} \sum_{i=2}^{n} X_i^2\right)^{1/2}$ is

(a) 't' with n degrees of freedom

(b) 't' with (n – 1) degrees of freedom

(c) Non-central 't' with n degrees of freedom

(d) None of these

(iii) A non-central chi-square distribution is a

(a) Weighted sum of chi-square variables with weight as Poisson probabilities.

(b) Weighted sum of Poisson variables with weight as chi-square probabilities.

(c) Compound distribution of Poisson and chi-square distributions.

(d) (a) and (c) but not (b).

(iv) The probability mass function of the Poisson Bionomial distribution is

(a) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{n\,r}{x} p^x q^{n\,r-x} \frac{\lambda^r}{r!}$

(b) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{n\,r}{x} p^{-x} q^{n\,r-x} \frac{\lambda^r}{r!}$

(c) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{n\,r}{x} p^x q^{-n\,r-x} \frac{\lambda^r}{r!}$

(d) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{n\,r}{x} p^x q^{n\,r-x} \frac{\lambda^{-r}}{r!}$

(v) The probability generating function of the Poisson distribution is

(a) $G(Z) = e^{\lambda + \lambda e^{-m + mz}}$ (b) $G(Z) = e^{-\lambda + \lambda e^{-m + mz}}$

(c) $G(Z) = e^{\lambda + \lambda e^{m + mz}}$ (d) $G(Z) = e^{-\lambda + \lambda e^{m + mz}}$

(vi) The probability generating function of the Poisson Negative Binomial distribution is

(a) $G(Z) = e^{-\lambda - \lambda (q - pz)^{-n}}$ (b) $G(Z) = e^{\lambda - \lambda (q - pz)^{-n}}$

(c) $G(Z) = e^{-\lambda + \lambda (q - pz)^{-n}}$ (d) $G(Z) = e^{-\lambda + \lambda (q + pz)^{-n}}$

(vii) The recurrence relation for the probability for the Neyman type-A distribution is

(a) $P_{r+1} = \frac{u_1' e^{-m}}{r + 1} \sum_{j=0}^{r} \frac{m^j}{j!} p_{r-j}$ (b) $P_{r+1} = \frac{u_1' e^{-m}}{r - 1} \sum_{j=0}^{r} \frac{m^j}{j!} p_{r-j}$

(c) $P_{r+1} = \frac{u_1' e^{-m}}{r + 1} \sum_{j=0}^{r} \frac{m^j}{j} p_{r-j}$ (d) $P_{r+1} = \frac{u_1' e^{-m}}{r + 1} \sum_{j=0}^{r} \frac{m^j}{j!} p_{r-1}$

(viii) Which one of the following statement is not true ?

(a) When 'v=1', student's t distribution tends to Weibull distribution.

(b) When 'v'=1', student's t distribution tends to Cauchy distribution.

(c) The sampling distribution of F-statistic does not involve any population parameter.

(d) The non-central Chi-square distribution is the mixture of central Chi-square distribution and Poisson distribution.

(ix) Which one of the following statement is not true ?

(a) For Poisson Binomial distribution mean is less than variance.

(b) For Poisson Pascal distribution means is less than variance.

(c) Neyman type-A distribution tends to Neyman type-B distribution.

(d) Neyman type-B distribuition tends to Neyman type-A distribution.

(x) The moment generating function of non-central chi-square distribution is

(a) $M_{\chi}2(t) = (1 - 2\,t)^{-n/2} \exp\left(\frac{2t\lambda}{1-2t}\right) \forall\, t < 1/2$

(b) $M_{\chi}2(t) = (1 - 2\,t)^{-1/2} \exp\left(\frac{2t\lambda}{1-2t}\right) \forall\, t > 1/2$

(c) $M_{\chi}2(t) = (1 - 2\,t)^{-n/2} \exp\left(\frac{2t\lambda}{1-2t}\right) \forall\, t > 1/2$

(d) $M_{\chi}2(t) = (1 - 2\,t)^{-n/2} \exp\left(\frac{2t\lambda}{1-2t}\right) \forall\, t \neq 1/2$

(xi) If X is a non-central chi-square variate with degrees 5 and non-centrality parameter δ is also 5, then E(X) and V(X) are respectively

(a) (10, 30) (b) (15, 50)

(c) (5, 10) (d) None of these

(xii) If a statistics t follows Student's t distribution with degrees of freedom n, then t^2 follows :

(a) Student's distribution with n^2 degrees of freedom.

(b) Snedecor's distribution with (1, n) degrees of freedom.

(c) Snedecor's distribution with (n, 1) degrees of freedom.

(d) None of these.

(xiii) Descending factorial cumulant generating function H(t) is defined as

(a) $\text{Log } E(1+t)^{x}$ (b) $\text{Ln } E\,(1+t)^{x}$

(c) $\text{Exp}(E(1+t)^{x})$ (d) $\text{Log } E\,(1-t)^{-x}$

(xiv) If a random sample of size 5 is taken from Uniform distribution, then the probability density function of the sample median is

(a) the probability density function of the third order statistics.

(b) the probability density function of the fifth order statistics.

(c) the probability density function of the first order statistics.

(d) None of these.