Seat No. : _______________

15G-105

May-2015

M.Sc., Sem.-II

410 : Mathematics
(Partial Differential Equations)

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Attempt any **one**. **7**

(i) Find the general integral of $(x^2 + 3y^2 + 3z^2)p - 2xyq + 2xz = 0$.

(ii) Verify that the Pfaffian differential equation

$$(6x + yz)\,dx + (xz - 2y)dy + (xy + 2z)dz = 0$$

is integrable and find its integral.

(b) Attempt any **two**. **4**

(i) If $\vec{X} \cdot \text{curl}\,\vec{X} = 0$ where $\vec{X} = (P, Q, R)$ and μ is an arbitrary differentiable function of x, y and z, prove that $\mu\vec{X}.\text{curl}\,(\mu\vec{X}) = 0$.

(ii) Solve : $yzdx + xzdy + xydz = 0$.

(iii) Show that the equations

$$p^2 + q^2 - 1 = 0,\ (p^2 + q^2)x - pz = 0$$

are compatible and find the one parameter family of common solutions.

(c) Answer very briefly. **3**

(i) Form a partial differential equation by eliminating the arbitrary function F from $F(z - xy, x^2 + y^2) = 0$.

(ii) Find the envelope of $(x - a)^2 + (y - 2a)^2 + z^2 = 1$.

(iii) What is the integral of $ydx + xdy + 2zdz = 0$?

2. (a) Attempt any **one**. **7**

(i) Find a complete integrals of $x^2p^2 + y^2q^2 - 4 = 0$ by Charpit's method.

(ii) Find the integral surface of the differential equation

$$p^2x + pqy - 2pz - x = 0,$$

passing through the line $y = 1$, $x = z$.

(b) Attempt any **two**. **4**

(i) By Jacobi's method, solve the equation $xu_x + yu_y - u_z^2 = 0$.

(ii) Solve $z_x + z_y = z^2$ with the initial condition $z(x, 0) = f(x)$.

(iii) Find a complete integral of the equation $p^2 = qz$.

(c) Answer very briefly. **3**

(i) Define complete integral of the p.d.e. $f(x, y, z, u_x, u_y, u_z) = 0$.

(ii) What is a complete integral of the equation $z = px + qy + p - q$?

(iii) What is a complete integral of the equation $p + q = pq$?

3. (a) Attempt any **one**. **7**

(i) Reduce the equation $u_{xx} - y^4 u_{yy} = 2y^3 u_y$ to a canonical form and solve it.

(ii) Solve the one-dimensional wave equation

$$y_{xx} = \frac{1}{c^2} y_{tt}, \quad -\infty < x < \infty, \ t > 0$$

with the initial conditions

$$y(x, 0) = f(x), \ y_t(x, 0) = g(x), \ -\infty < x < \infty$$

where $f \in C^2$ and $g \in C^1$.

(b) Attempt any **two**. **4**

(i) Find the characteristic strips of the equation $z + px + qy = 1 + pqx^2y^2$ passing through the initial data curve $C : x_0 = s, \ y_0 = 1, \ z_0 = -s$.

(ii) Classify the equation $xu_{xx} + 2\sqrt{xy}u_{xy} + yu_{yy} = u_x$.

(iii) How many possible solutions of the equation $z = p^2 - q^2$ which passes through the curve $C : x_0 = s, \ y_0 = 0, \ z_0 = -\frac{1}{4} s^2$? Justify.

(c) Answer very briefly. **3**

(i) Of which type is the equation $u_{xx} + 2u_{xy} + 17u_{yy} = 0$?

(ii) What is the canonical form of the elliptic type second order semi-linear p.d.e. ?

(iii) Give an example of a second order semi-linear p.d.e. which is of the parabolic type.

4. (a) Attempt any **one**. **7**

(i) Suppose that $u(x, y)$ is harmonic in a bounded domain D and continuous in $\bar{D} = D \cup B$. Prove that u attains its maximum on the boundary B of D.

(ii) Prove that the solution for the Dirichlet problem for a circle of radius a is given by the Poisson integral formula.

(b) Attempt any **two**. **4**

(i) Prove that the solution of the Dirichlet problem, if it exists, in unique.

(ii) State the Dirichlet problem for a rectangle.

(iii) Show that the solution of the Dirichlet problem is stable.

(c) Answer very briefly. **3**

(i) State the Dirichlet problem for the upper half plane.

(ii) What is a Cauchy problem ?

(iii) What are the Hadamard's conditions for a well posed problem ?

5. (a) Attempt any **one**. **7**

(i) Solve the following problem :

$$u_t = ku_{xx}, \qquad 0 < x < l, t > 0,$$

$$u(0, t) = u(l, t) = 0, \qquad t > 0,$$

$$u(x, 0) = f(x), \qquad 0 \leq x \leq l.$$

(ii) State and prove Harnack's theorem.

(b) Attempt any **two**. **4**

(i) State the heat conduction problem for a infinite rod.

(ii) Show that the surface $x^2 + y^2 + z^2 = c$, $c > 0$, can form an equipotential family of surfaces.

(iii) Let D be a bounded domain in $\mathbb{R}^2$, bounded by a smooth closed curve B. Let $\{u_n\}$ be a sequence of functions each of which is continuous on $\bar{D} = D \cup B$ and harmonic in D. If $\{u_n\}$ converges uniformly on B, prove that $\{u_n\}$ converge uniformly on $\bar{D}$.

(c) Answer very briefly. **3**

(i) State the Neumann problem for the upper half plane.

(ii) State the Neumann problem for a circle of radius a.

(iii) Show that $\phi = (x^2 + y^2 + z^2)^{-1/2}$ satisfies the three-dimensional Laplace's equation in any domain that does not contain the origin.
