2504E411

Candidate's Seat No :________

IM.Sc. (AIML) Sem.-4 Examination

CC-213

Vector Calculus

April-2025

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Find the unit tangent and length for $0 \le t \le \pi$ of the curve $\bar{r}(t) = (1 + 2\cos t)\,\hat{\imath} + (2\sin t)\hat{\jmath} + \sqrt{3}t\hat{k}$. **(07)**

(b) Find the unit normal and curvature of the curve $\bar{r}(t) = 2e^t \sin t\,\hat{\imath} + 2\,e^t \cos t\,\hat{\jmath} + 3\hat{k}$. **(07)**

OR

(a) Find the velocity, acceleration, and speed of particle with position vector $\bar{r}(t) = (2(\ln(t+1))\,\hat{\imath} + t^2\hat{\jmath} + \frac{t^2}{2}\hat{k}$ for $t = 1$. **(07)**

(b) Find the torsion τ for the curve $\bar{r}(t) = (a\cos t)\,\hat{\imath} + (a\sin t)\hat{\jmath} + bt\hat{k}$, where $a, b \ge 0$, $a^2 + b^2 \ne 0$. **(07)**

Q.2 **(a)** Find the directional derivative of $\phi = x^2y^2z^2$ at $(1,1,-1)$ in the direction of the tangent to the curve $x = e^t, y = 2\sin t + 1, z = t - \cos t$ at $t = 0$. **(07)**

(b) Find rate of fluid flow loss per unit volume of $\bar{r} = \sin x\,\hat{\imath} + \cos y\,\hat{\jmath} + z^2\hat{k}$ at $(0,0,-1)$. **(07)**

OR

(a) In what direction the directional derivative of the curve $\phi = x^2 + y^2 - z$ at point $(1,2,2)$ is maximum? Also find maximum directional derivative. **(07)**

(b) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z = 47$ at $(4,-3,2)$. **(07)**

Q.3 **(a)** Prove that the velocity given by $\bar{F} = (y+z)\hat{\imath} + (z+x)\hat{\jmath} + (x+y)\hat{k}$ is irrotational and find its scalar potential. **(07)**

(b) Integrate $\bar{F} = 3yz\hat{\imath} - zx\hat{\jmath} - xy\hat{k}$ over the curve $\bar{r}(t) = t\hat{\imath} + t^2\hat{\jmath} + t^3\hat{k}, 0 \le t \le 2$ **(07)**

OR

(a) Evaluate $\int_C \frac{x+y^2}{\sqrt{1+x^2}}\,ds$ over the curve $C: y = \frac{x^2}{2}$ from $(1,\frac{1}{2})$ to $(0,0)$ **(07)**

(b) Find the line integral along the straight line-segments joining the points (0,0,0) to (1,0,0), then to (1,1,0) and finally to (1,1,1) of the function $\bar{F} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$ **(07)**

Q.4 **(a)** How many distinguishable permutations of the letters in the word BANANA are there? **(07)**

(b) Using Pigeonhole Principle show that if any five numbers from 1 to 8 are chosen, then two of them will add up to 9. **(07)**

OR

(a) Suppose we remove a square from a standard 8×8 chessboard then by Principle of Mathematical Induction show that one can tile the 63 remaining squares of chessboard by L-shaped triominoes. **(07)**

(P.T.O)

(b) Solve the recurrence relations: **(07)**

(i) $a_n = -6a_{n-1} - 9a_{n-2}$ where $a_1 = 2.5, a_2 = 4.7$

(ii) $a_n = -3a_{n-1} - 2a_{n-2}$ with $a_1 = -2, a_2 = 4$

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Find the arc length of the curve $\bar{r}(t) = (1 + 3t^2)\hat{\imath} + (4 + 2t^3)\hat{\jmath}\,,\, 0 \le t \le 1.$

(2) Find the magnitude of the velocity and acceleration of a particle which moves along the curve $x = 2\sin 3t\,, y = 2\cos 3t\,, z = 8t$ at any time $t > 0$.

(3) Find the point on the curve $\bar{r}(t) = 12t\hat{\imath} + 5\sin t\,\hat{\jmath} + 5\cos t\,\hat{k}$ at a distance 13π units along the curve from the point (0,0,5) in the direction of increasing arc length.

(4) Find the unit vector normal to the surface $x^2y + 2xz^2 = 8$ at the point (1,0,2).

(5) Determine the constant a if vector $\bar{F} = (x - 5z)\hat{\imath} + (2y - 3x^2)\hat{\jmath} + (3x^2 + az)\hat{k}$ is solenoidal.

(6) If $\phi = x^2y + y^2x + z^2$ then find $grad\ \emptyset$ at (1,1,1).

(7) For $\bar{F} = x^2y\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$ find the curl at the point (1,0,2)

(8) Find the values of a and b such that the vector field $\bar{F} = (x + y + az)\hat{\imath} + (bx + 2y - z)\hat{\jmath} + (-x + cy + 2z)\hat{k}$ is irrotational.

(9) Evaluate $\int_C (x + y)\, ds$ where C is the straight-line segment $x = t\,, y = (1 - t), z = 0$ from (0,1,0) to (1,0,0).

(10) Thirty cars were assembled in a factory. The options available were a radio, an air conditioner and white wall tires. It is known that 15 of the cars have radios, 8 of them have air conditioners and 6 of them have white-wall tires. Moreover, 3 of them have all three options. Determine at least how many cars do not have any options at all.

(11) Show that if seven colors are used to paint 50 bicycles, at least 8 bicycles will be the same color.

(12) How many different seven member committees can be formed each containing 3 women from an available set of 20 women and 4 men from an available set of 30 men?

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