

## IM.Sc. (DS) (NEP) Sem.-4 Examination

DSC-C-DTSC-242-T

Linear Algebra for D.S.

April-2025

[Max. Marks : 50]

Time : 2-00 Hours]

**Instructions:** All questions are compulsory. Use of non-programmable scientific calculator is allowed.

**Q.1 (a)** Find  $(v \cdot u)$ ,  $|v|$ ,  $|u|$  and angle between  $v = i - 2j + \sqrt{3}k$ ,  $u = -i + 2j - \sqrt{3}k$ . (05)

**(b)** Let  $S = \{(1,2,1), (1,1,-1), (4,5,-2)\}$ . Determine which of the following vectors are in  $[S] = \text{span}(S)$ . (05)

i)  $(1,1,0)$

ii)  $(2,-1,-8)$

OR

**(a)** Find the measure of the angle between the diagonals of the rectangle whose vertices are  $A = (1,0)$ ,  $B = (0,3)$ ,  $C = (3,4)$  and  $D = (4,1)$ . (05)

**(b)** Which of the following subsets of the  $R^4$  are vector space for coordinate wise addition and scalar multiplication? The set of all vectors  $(x_1, x_2, x_3, x_4) \in R^4$  such that (05)

i)  $x_1 = 1$

ii)  $x_3 \geq 0$

**Q.2 (a)** Given  $\alpha_1 = (1,1,1)$ ,  $\alpha_2 = (1,2,3)$  and  $\alpha_3 = (1,3,4) \in R^3$ . Find an  $\alpha \in R^3$  such that  $\langle \alpha, \alpha_1 \rangle = 7$ ,  $\langle \alpha, \alpha_2 \rangle = 16$ ,  $\langle \alpha, \alpha_3 \rangle = 22$ . (05)

**(b)** Let  $B = \{1 - t^2, t - t^2, 2 - 2t + t^2\}$  be an ordered basis for vector space of polynomials of degree  $\leq 2$ . Find the co-ordinate vector  $f(x) = -5x^2 + 3x + 6$  relative to basis  $B$ . (05)

OR

**(a)** Identify the conditions on  $a$ ,  $b$  and  $c$  so that the following system is consistent. (05)

$$x + 2y + 3z = a$$

$$2x + 5y + 3z = b$$

$$x + 8y = c$$

**(b)** Find dimension and basis of row space of following matrix: (05)

$$\begin{bmatrix} 5 & 4 & 1 \\ 11 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

- Q.3 (a)** Find Rank, Kernel and Nullity of the following linear transformation (05)

$$T: R^2 \rightarrow R^3, T(x, y) = (x + y, x - y, y)$$

- (b)** Let  $T: R^2 \rightarrow R^2$  have a linear map. Suppose that  $L(1,1) = (1, 4)$  and  $L(2, -1) = (-2, 3)$  (05)  
then find  $L(3, -1)$ .

**OR**

- (a)** Let  $T: R^3 \rightarrow R^3, T(x, y, z) = (x - 2y, 2x + 3z, -2z + x)$  and  $S: R^3 \rightarrow R^3,$  (05)  
 $S(x, y, z) = (4x, x - z, -z)$  are linear transformations then find transformation matrix  
[ $T \circ S$ ].

- (b)** Let  $T$  be a linear transformation from  $R^3 \rightarrow R^3$  defined by (05)  
 $T(x, y, z) = (2x + y, 2y - 3x, x - z)$ , find  $T^{-1}$  if exist.

- Q.4 (a)** Define Diagonalization of a matrix. Check whether the matrix is diagonalizable or not. (05)

$$A = \begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$$

- (b)** Describe the conic whose equation is  $x^2 + y^2 + 6x - 10y + 18 = 0$ . (05)  
Give its equation in the translated coordinate system.

**OR**

- (a)** Determine the nature (value class), index and signature of the following quadratic form: (05)  
 $x_1^2 + 5x_2^2 + x_3^2 + 2x_2x_3 + 6x_3x_1 + 2x_1x_2$

- (b)** Define Singular Value Decomposition. Compute the singular values of the matrix (05)

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

- Q.5** Do as directed. Attempt any **TEN** out of **TWELVE**: (Each carries 01 mark) (10)

**Q-1 to 9 – True or False**

- (1) If  $A(1,5, -4)$  and  $B(2,3,1)$ , then  $\overline{AB} = 2i + 6j - 4k$ .
- (2) The vectors  $(1,1)$  and  $(-1, 2)$  form a basis of  $R^2$ .
- (3) The vectors  $(1,1,1)$  and  $(0,1, -2)$  are linearly independent.
- (4) The vector  $v = (0,0,4)$  has length=2 and directions= $-k$ .
- (5) The following non-homogeneous systems of linear equation has unique solution.

$$3x + y + 2z = 0$$

$$x - 2y + 3z = 0$$

$$x + 5y - 4z = 0$$

- (6) The following function  $T: R^2 \rightarrow R^3, T(x, y) = (x - y, y + 3, z)$  is linear.
- (7) The transformation  $T(x, y) = (0, x)$  is an orthogonal projection on the x-axis.
- (8) Let  $T: V \rightarrow W$  be a linear map then  $T$  is one to one if  $Ker(T) = 0$ .
- (9) The transformation  $T(x, y) = (x, y + x)$  gives vertical shearing of an image.
- (10) Define geometric interpretation of Eigenvalues and Eigenvectors.
- (11) State (only) Cayley-Hamilton Theorem.
- (12) If  $A$  is a singular matrix of order  $3 \times 3$  whose eigenvalues are 5 and 7, then what will be the third eigen value.

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