2404E373 Candidate's Seat No :________

IM.Sc. (AIML) (NEP) Sem.-4 Examination

DSC-C-AIML-242-T

Linear Algebra for M. L.

April-2025

Time : 2-00 Hours] **[Max. Marks : 50**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Find $(v.u)$, $|v|$, $|u|$ and angle between $v = i - 2j + \sqrt{3}k, u = -i + 2j - \sqrt{3}k$. **(05)**

(b) Let $S = \{(1,2,1), (1,1,-1), (4,5,-2)\}$. Determine which of the following vectors are in $[S] = span(S)$. **(05)**

i) $(1,1,0)$

ii) $(2,-1,-8)$

OR

(a) Find the measure of the angle between the diagonals of the rectangle whose vertices are $A = (1,0), B = (0,3), C = (3,4)$ and $D = (4,1)$. **(05)**

(b) Which of the following subsets of the R^4 are vector space for coordinate wise addition and scalar multiplication? The set of all vectors $(x_1, x_2, x_3, x_4) \in R^4$ such that **(05)**

i) $x_1 = 1$

ii) $x_3 \geq 0$

Q.2 **(a)** Given $\alpha_1 = (1,1,1), \alpha_2 = (1,2,3)$ and $\alpha_3 = (1,3,4) \in R^3$. Find an $\alpha \in R^3$ such that $\langle \alpha, \alpha_1 \rangle = 7, \langle \alpha, \alpha_2 \rangle = 16, \langle \alpha, \alpha_3 \rangle = 22$. **(05)**

(b) Let $B = \{1 - t^2, t - t^2, 2 - 2t + t^2\}$ be an ordered basis for vector space of polynomials of degree ≤ 2. Find the co-ordinate vector $f(x) = -5x^2 + 3x + 6$ relative to basis B. **(05)**

OR

(a) Identify the conditions on a, b and c so that the following system is consistent. **(05)**

$$x + 2y + 3z = a$$

$$2x + 5y + 3z = b$$

$$x + 8y = c$$

(b) Find dimension and basis of row space of following matrix: **(05)**

$$\begin{bmatrix} 5 & 4 & 1 \\ 11 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

(P.T.O)

Q.3 **(a)** Find Rank, Kernal and Nullity of the following liner transformation **(05)**

$$T: R^2 \to R^3, T(x,y) = (x+y, x-y, y)$$

(b) Let $T: R^2 \to R^2$ have a linear map. Suppose that $L(1,1) = (1,4)$ and $L(2,-1) = (-2,3)$ then find $L(3,-1)$. **(05)**

OR

(a) Let $T: R^3 \to R^3, T(x,y,z) = (x-2y, 2x+3z, -2z+x)$ and $S: R^3 \to R^3$, $S(x,y,z) = (4x, x-z, -z)$ are linear transformations then find transformation matrix $[ToS]$. **(05)**

(b) Let T be a liner transformation form $R^3 \to R^3$ defined by $T(x,y,z) = (2x+y, 2y-3x, x-z)$, find T^{-1} if exist. **(05)**

Q.4 **(a)** Define Diagonalization of a matrix. Check whether the matrix is diagonalizable or not. **(05)**

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(b) Describe the conic whose equation is $9x^2 + 4y^2 - 36x - 24y + 36 = 0$. Give its equation in the translated coordinate system. **(05)**

OR

(a) Define Algebraic and Geometric Multiplicity of the matrix. Determine AM and GM of each eigenvalue of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ **(05)**

(b) Define Singular Value Decomposition. Find a SVD of the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$. **(05)**

Q.5 Do as directed. Attempt any **TEN** out of **TWELVE:** (Each carries 01 mark) **(10)**

Q-1 to 9 – True or False

(1) If $A(1,5,-4)$ and $B(2,3,1)$, then $\overrightarrow{AB} = 2i + 6j - 4k$.

(2) The vectors $(1,1)$ and $(-1,2)$ form a basis of R^2.

(3) The vectors $(1,1,1)$ and $(0,1,-2)$ are linearly independent.

(4) The vector $v = (0,0,4)$ has length=2 and directions=$-k$.

(5) The following non-homogeneous systems of linear equation has unique solution.

$$3x + y + 2z = 0$$
$$x - 2y + 3z = 0$$
$$x + 5y - 4z = 0$$

(6) The following function $T: R^2 \to R^3, T(x, y) = (x + y, y + 3, z + 2)$ is linear

(7) The transformation $T(x, y) = (x, x)$ is an orthogonal projection on the x-axis.

(8) Let $T: V \to W$ be a linear map then T is one to one if $Ker(T) = 0$.

(9) The transformation $T(x, y) = (x, y + 2)$ gives vertical shearing of an image.

(10) Define: Eigenvalues and Eigenvectors.

(11) Define: Cayley-Hamilton Theorem.

(12) If A is a singular matrix of order 3×3 whose eigenvalues are 2 and 3, then what will be the third eigen value.

_________________****____________****______________