Seat No. : _______________

AK-119

April-2025

B.Sc., Sem.-IV

DSC-C-MAT-241T : Mathematics (Major)

(Linear Algebra – II)

Time : 2:00 Hours] **[Max. Marks : 50**

Instructions : (1) **All** questions are compulsory.
(2) Write the question number in your answer book as shown in the question paper.
(3) The figure to the right indicates marks of the question.

1\. (a) Evaluate $D = \begin{vmatrix} 2 & 5 & 7 & 8 \\ 3 & 6 & 2 & 4 \\ 3 & 5 & 7 & 6 \\ 1 & 6 & 9 & 7 \end{vmatrix}$ using Vedic method (Urdhva Tiryabham). **5**

(b) Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by Vedic method. **5**

OR

1\. (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 2 \\ 3 & 1 & 4 \end{bmatrix}$ by using "Urdhva Tiryak" method. **5**

(b) Solve the following system using Parävartya rule :
$2x + 3y = 8$, $4x + 5y = 14$. **5**

2\. (a) State and prove the dual basis existence theorem. **5**

(b) Let W_1 and W_2 be two subspaces of a finite dimensional vector space V. Prove that $(W_1 + W_2)^\circ = W_1^\circ \cap W_2^\circ$. **5**

OR

2\. (a) Let V be a finite dimensional vector space over the field F and let W be a subspace of V. Then Prove that dim W + dim W° = dim V. **5**

(b) Let $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in R^2. Let $<\bullet, \bullet> : R^2 \times R^2 \to R$ be a function defined as $\langle x, y\rangle = x_1y_1 - x_2y_1 - x_1y_2 + 4x_2y_2$ then, check whether <, > is an inner product on R^2 or not. **5**

3. (a) State and prove Cauchy-Schwartz's inequality. **5**

(b) Using Cramer's Rule (If applicable), Solve the system of linear equations : $x + 2y + 3z = 3$, $2x - z = 4$, $4x + 2y + 2z = 5$. **5**

OR

3. (a) Let $A = (a_{ij}) \in M_n$ be such that two rows are equal. Prove that det A = 0. **5**

(b) Let W be a subspace of a finite dimensional inner product space V. Prove that $V = W \oplus W^{\perp}$. **5**

4. (a) State and prove Cayley-Hamilton Theorem. **5**

(b) Find the Eigen values and Eigen vectors of the following matrix : **5**

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

OR

4. (a) Prove that Eigen vectors corresponding to distinct Eigen values are linearly independent. **5**

(b) Find A^{-1} of $A = \begin{pmatrix} 3 & 1 & -1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ using Cayley-Hamilton Theorem. **5**

5. Answer in brief : (Any **ten**) **10**

(i) Let $T : R^3 \to R^2$ and $S : R^2 \to R^2$ be two linear maps defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, x_3)$ and $S(x_1, x_2) = (x_2, x_1)$ then determine ST.

(ii) Les S be the set of all symmetric matrices of order n with entries in R. Find dim S.

(iii) Let $A = \begin{bmatrix} 1 & 5 & 6 \\ 0 & 6 & 8 \\ 0 & 0 & 7 \end{bmatrix}$. Find Eigen values of A.

(iv) Let $V = R^3$, $S = \{(1, 0, 0), (0, 1, 0)]$. What is $S^{\perp}$?

(v) Define : Determinant of a square matrix.

(vi) Define : Operator equation.

(vii) Define : Eigen value and Eigen vector.

(viii) Define : Non-singular linear transformation.

(ix) What are the Eigen values of $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$?

(x) What is the sum of Eigen values of the following matrix ?

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

(xi) Define : Similar matrices.

(xii) If $A = \begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{pmatrix}$ is a symmetric matrix or skew symmetric matrix ? Justify your answer.
