2304E327

Candidate's Seat No :_________

IM.Sc. (DS) Sem.-4 Examination

CC-211

Linear Algebra

April-2025

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Let R^3 have the Euclidean inner product. For which values of k are u and v orthogonal? **(07)**
(i) $u = (k, k, 1), v = (k, 5, 6)$
(ii) $u = (1, k, -3), v = (2, -5, 4)$

(b) Evaluate the following if $u = (0,2,3,1), v = (2,0,-1,-1),\ w = (-3,-1,-2,0)$ **(07)**

(i) $||u + v||$
(ii) $||2u + 3v + 4w||$

OR

(a) Find a condition on a, b, c so that vector $v = (a, b, c)$ is in span $\{v_1, v_2, v_3\}$. **(07)**
Where, $v_1 = (2,1,0), v_2 = (1,-1,2), v_3 = (0,3,-4)$.

(b) Check whether the following set of matrices in M_{22} is linearly dependent or not. **(07)**
$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 6 \\ 4 & 6 \end{bmatrix}$

Q.2 **(a)** Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis vectors $u_1 = (1,2,1), u_2 = (1,0,1), u_3 = (3,1,0)$ into an orthogonal basis. **(07)**

(b) Determine the dimension and a basis for the solution space of the systems: **(07)**
$x_1 - 3x_2 + x_3 = 0$
$2x_1 - 6x_2 + 2x_3 = 0$
$3x_1 - 9x_2 + 3x_3 = 0$

OR

(a) Find an orthonormal basis for R^3 containing the vectors $(2,-2,1)$ and $(2,1,-2)$ using Euclidean inner product. **(07)**

(b) Consider the bases $S_1 = \{u_1, u_2\}$ and $S_2 = \{v_1, v_2\}$ where $u_1 = (1,-2)$, $u_2 = (3,-4), v_1 = (1,3), v_2 = (3,8)$. Find the transition matrix from S_2 to S_1. **(07)**

Q.3 **(a)** Which of the following are linear transformations? Justify. **(07)**

(i) $T: R^2 \rightarrow R^3$, where $T(x, y) = (x + 1, 2y, x + y)$
(ii) $T: M_{22} \rightarrow M_{22}$, where $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} b & c - d \\ c + d & 2a \end{bmatrix}$

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(b) Let T be a linear transformation form $R^3 \rightarrow R^3$ defined by $T(x, y, z) = (2x + y, 2y - 3x, x - z)$, find T^{-1} if exist. **(07)**

OR

(a) Consider the basis $S = \{v_1, v_2\}$ for R^2, where $v_1 = (-2,1)$, $v_2 = (1,3)$ and let $T: R^2 \rightarrow R^3$ be the linear transformation such that $T(v_1) = (-1,2,0)$, $T(v_2) = (0, -3,5)$. Find a formula for $T(x_1, x_2)$ and use that formula to find $T(2, -3)$. **(07)**

(b) Let $T: R^3 \rightarrow R^3$ be the linear transformation defined by **(07)**

$$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$$

(i) Find a basis and the dimension for the range of T.
(ii) Find a basis and the dimension for the kernel of T.

Q.4 **(a)** Determine algebraic and geometric multiplicity of the following matrix: **(07)**

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Find A^4, where $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$. **(07)**

OR

(a) Verify Cayley-Hamilton theorem for the matrix A and hence, find A^{-1} **(07)**

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$$

(b) Determine the nature (value class), index and signature of the following quadratic from $x_1^2 + 4x_2^2 + x_3^2 - 4x_1x_2 + 2x_3x_1 - 4x_2x_3$ **(07)**

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Define: Projection Operators

(2) Check whether the following set is subspace of R^3 or not.

$$W = \{(x, y, z) | y = x + z + 1\}$$

(3) Check whether the following function is linear transformation or not. $T: R^2 \rightarrow R^2$, where $T(x, y) = (x + 2y, 3x - y)$

(4) Determine whether multiplication by A is one-one or not.

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -4 \\ -3 & 6 \end{bmatrix}$$

(5) Let $T: R^3 \to R^3$ be the orthogonal projection of R^3 on to the xy-plane. Show that $T \circ T = T$.

(6) Let $u = (u_1, u_2), v = (v_1, v_2)$. Find a matrix that generates the following inner product. $< u, v >= 4u_1v_1 + 6u_2v_2$

(7) Find $d(u, v)$ if $u = (5,4), v = (2, -6)$ and weighted Euclidean inner product is $< u, v >= 3u_1v_1 + 2u_2v_2$ where $u = (u_1, u_2), v = (v_1, v_2)$.

(8) Write down the quadratic form corresponding to following matrix:

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

(9) Show that the set of vectors $u_1 = \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right), u_2 = \left(-\frac{1}{2}, \frac{1}{2}, 0\right), u_3 = \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ is orthogonal with respect to the Euclidean inner product on R^3.

(10) Find the orthogonal projection of $u = (1, -2,3)$ along $v = (1,2,1)$ in R^3 with respect to the Euclidean inner product.

(11) Find the eigen values of the given matrix:

$$\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

(12) Find the cosine of the angle between u and v if R^2 have the Euclidean inner product. $u = (1, -3), v = (2,4)$

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