# 2304E328

Candidate's Seat No :_________

**IM.Sc. (AIML) Sem.-4 Examination**

**CC-211**

**Linear Algebra**

**April-2025**

**Time : 2-30 Hours]** **[Max. Marks : 70**

**Instructions:** All questions are compulsory. Use of non-programmable scientific calculator is allowed.

**Q.1** **(a)** Find a condition on $a, b, c$ so that vector $v = (a, b, c)$ is in span $\{v_1, v_2, v_3\}$. Where, $v_1 = (2,1,0), v_2 = (1,-1,2), v_3 = (0,3,-4)$. **(07)**

**(b)** Evaluate the following if $u = (0,2,3,1), v = (2,0,-1,-1),\ w = (-3,-1,-2,0)$ **(07)**

**(i)** $||u + v||$

**(ii)** **(ii)** $||2u + 3v + 4w||$

**OR**

**(a)** Let $R^3$ have the Euclidean inner product. For which values of $k$ are $u$ and $v$ orthogonal? **(07)**

**(i)** $u = (k, k, 1), v = (k, 5, 6)$

**(ii)** $u = (1, k, -3), v = (2, -5, 4)$

**(b)** Check whether the following set of matrices in $M_{22}$ is linearly dependent or not. **(07)**

$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 6 \\ 4 & 6 \end{bmatrix}$

**Q.2** **(a)** Consider the bases $\boldsymbol{S_1} = \{\boldsymbol{u_1}, \boldsymbol{u_2}\}$ and $\boldsymbol{S_2} = \{\boldsymbol{v_1}, \boldsymbol{v_2}\}$ where $\boldsymbol{u_1} = (\mathbf{1}, -\mathbf{2})$, $\boldsymbol{u_2} = (\mathbf{3}, -\mathbf{4}), \boldsymbol{v_1} = (\mathbf{1}, \mathbf{3}), \boldsymbol{v_2} = (\mathbf{3}, \mathbf{8})$. Find the transition matrix from $\boldsymbol{S_2}$ to $\boldsymbol{S_1}$. **(07)**

**(b)** Find an orthonormal basis for $R^3$ containing the vectors $(2,-2,1)$ and $(2,1,-2)$ using Euclidean inner product. **(07)**

**OR**

**(a)** Determine the dimension and a basis for the solution space of the systems: **(07)**

$x_1 - 3x_2 + x_3 = 0$

$2x_1 - 6x_2 + 2x_3 = 0$

$3x_1 - 9x_2 + 3x_3 = 0$

**(b)** Let $R^3$ have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis vectors $u_1 = (1,2,1), u_2 = (1,0,1), u_3 = (3,1,0)$ into an orthogonal basis. **(07)**

**Q.3** **(a)** Which of the following are linear transformations? Justify. **(07)**

**(i)** $T: R^2 \to R^3$, where $T(x, y) = (x + 1, 2y, x + y)$

**(ii)** $T: M_{22} \to M_{22}$, where $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} b & c - d \\ c + d & 2a \end{bmatrix}$



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**(b)** Let $\boldsymbol{T}: \boldsymbol{R}^3 \to \boldsymbol{R}^3$ be a multiplication of $\boldsymbol{A}$, if $\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ then find $\boldsymbol{T}^{-1}\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right)$. **(07)**

**OR**

**(a)** Let $\boldsymbol{T}: \boldsymbol{R}^3 \to \boldsymbol{R}^3$ be the linear transformation defined by **(07)**

$$\boldsymbol{T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)}$$

**(i)** Find a basis and the dimension for the range of $\boldsymbol{T}$.
**(ii)** Find a basis and the dimension for the kernel of $\boldsymbol{T}$.
**(iii)** Verify the dimension theorem.

**(b)** Consider the basis $S = \{v_1, v_2\}$ for $R^2$, where $v_1 = (-2,1)$, $v_2 = (1,3)$ and let $T: R^2 \to R^3$ be the linear transformation such that $T(v_1) = (-1,2,0)$, $T(v_2) = (0,-3,5)$. Find a formula for $T(x_1, x_2)$ and use that formula to find $T(2,-3)$. **(07)**

**Q.4** **(a)** Determine algebraic and geometric multiplicity of the following matrix: **(07)**

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

**(b)** Find $A^4$, where $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$. **(07)**

**OR**

**(a)** Verify Cayley-Hamilton theorem for the matrix $A$ and hence, find $A^{-1}$ and $A^4$. **(07)**

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$$

**(b)** Determine the nature (value class), index and signature of the following quadratic from **(07)**
$x_1{}^2 + 4x_2{}^2 + x_3{}^2 - 4x_1x_2 + 2x_3x_1 - 4x_2x_3$

**Q.5** Attempt any **SEVEN** out of **TWELVE:** **(14)**

**(1)** Check whether the following set is subspace of $R^3$ or not.
$W = \{(x, y, z) | y = x + z + 1\}$

**(2)** Define: Projection Operators

**(3)** Check whether the following function is linear transformation or not.
$T: R^2 \to R^2$, where $T(x, y) = (x + 2y, 3x - y)$

**(4)** Let $T: R^3 \to R^3$ be the orthogonal projection of $R^3$ on to the $xy$-plane.
Show that $T \circ T = T$.

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**(5)** Determine whether multiplication by $A$ is one-one or not.

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -4 \\ -3 & 6 \end{bmatrix}$$

**(6)** Let $u = (u_1, u_2), v = (v_1, v_2)$. Find a matrix that generates the following inner product.
$< u, v >= 4u_1v_1 + 6u_2v_2$

**(7)** Find $d(u, v)$ if $u = (5,4), v = (2, -6)$ and weighted Euclidean inner product is
$< u, v >= 3u_1v_1 + 2u_2v_2$ where $u = (u_1, u_2), v = (v_1, v_2)$.

**(8)** Find the cosine of the angle between $u$ and $v$ if $R^2$ have the Euclidean inner product.
$u = (1, -3), v = (2,4)$

**(9)** Show that the set of vectors $u_1 = \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right), u_2 = \left(-\frac{1}{2}, \frac{1}{2}, 0\right), u_3 = \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ is orthogonal with respect to the Euclidean inner product on $R^3$.

**(10)** Find the orthogonal projection of $u = (1, -2,3)$ along $v = (1,2,1)$ in $R^3$ with respect to the Euclidean inner product.

**(11)** Find the eigen values of the given matrix:

$$\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

**(12)** Write down the quadratic form corresponding to following matrix:

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

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