Seat No. : _____

AJ-103

April-2025

Integrated, M.Sc. (CA & IT), Sem.-IV

Matrix Algebra and Graph Theory (NEP)

Time : 2:00 Hours] **[Max. Marks : 50**

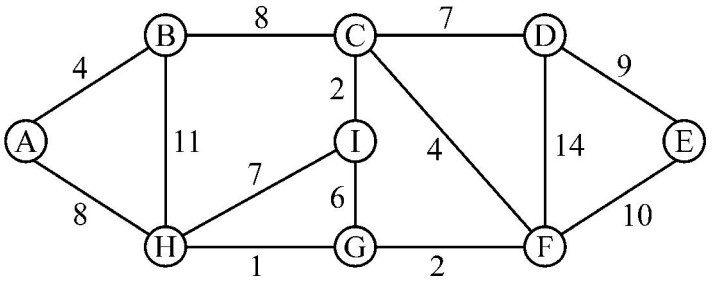
Instruction : Use of simple calculator is allowed.

1. Attempt any **two** : **10**

   (1) Let G be a k-regular graph such that k is an odd number. Prove that the number of vertices in G must be an even number.

   (2) Prove that there does not exist a party of eleven people where each person know exactly five of others.

   (3) Use Kruskal's algorithm to calculate the minimal spanning tree from the following graph with steps :



2. Attempt any **two** : **10**

   (1) Show that in a graph G if every vertex has degree 2 or more then G must contain a cycle.

   (2) Let G be a graph with n vertices and n-1 edges. Prove that G must contain either an isolated vertex or a vertex of degree 1.

   (3) Let G be a graph. Show that there exists a unique path between each pair of vertices in G if and only if G is a tree.

3. Attempt any **two** : **10**

   (1) Check whether a mapping $T : R^3 \to R^4$ defined by

   $T(x, y, z) = \left(\frac{4x - y + z}{3}, \frac{x + 4y - z}{3}, \frac{2z}{3}, \frac{-x + y + 4z}{3}\right)$ is a linear map or not ?

(2) Check whether a set U = {(3, 0, 2), (0, 3, 2), (3, 2, 0)} forms a Basis of R^3.

(3) Find a linear transformation T : $R^3 \to R^2$ such that

T(1, 0, 1) = (2, –3),

T(0, 1, 1) = (–1, 2),

T(1, 1, 0) = (–1, 1).

Also, find T (3/2, 1/5, –1/2).

4. Attempt any **two** : **10**

(1) How many permutations are possible using all letters of the word **"HEXAGON"** ? In dictionary order of these words, which place will this word occupy ?

(2) Let T : $R^3 \to R^3$ be a linear transformation defined by

T(x, y, z) = (–2x + 3y –z, x + 2y – 3z, 2x – 3y + 4z) . Find Ker(T).

(3) If $3\binom{n}{4} = 5\binom{n-1}{5}$, find n.

5. Attempt any **ten** : **10**

(1) Define linear transformation.

(2) Define kernel of a linear transformation.

(3) Define basis.

(4) Define tree.

(5) Give an example of bipartite graph which is not complete bipartite.

(6) Define adjacent vertices.

(7) Define connect graph.

(8) Write a vector (–3, 4) as a linear combination of (1, 2) and (3, 0).

(9) Does there exists a self-complementary graph on 11 vertices ? Justify your answer properly.

(10) Define Eulerian graph.

(11) Define path.