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0810N659

Candidate's Seat No :________

B.Sc. Sem.-6 (Rep) Examination

CC-307

Mathematics (New)

October-2025

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions : (i) All the questions are compulsory and carry equal marks.
(ii) Notations are usual everywhere.
(iii) The right hand side figures indicate marks of the question/sub-quest

Q.1

(a) If l, m, n are direction cosines of a line L then prove in usual notations that $l^2 + m^2 + n^2 = 1$. 7

(b) Find the equation of the plane passing through the lines of intersection of the planes $2x - y = 0$ and $3z - y = 0$ and perpendicular to the plane $4x + 5y - 3z = 8$. 7

OR

(a) Find the volume of the tetrahedron in terms of the coordinates of its vertices $A(x_1, y_1, z_1), B(x_2, y_2, z_2), C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$.

(b) A line makes an angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube then prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

Q.2

(a) Find the equation of a sphere having diameter extremities (i.e. diameter end points) (x_1, y_1, z_1) and (x_2, y_2, z_2). 7

(b) If the tangent plane to the sphere $x^2 + y^2 + z^2 = r^2$ makes intercepts a, b, c on the co-ordinate axes then show that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{r^2}$. 7

OR

(a) Derive the condition of orthogonality of the spheres

$$x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$$

and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$.

(b) Find the equation of the sphere through the points $(0,0,0), (0,1,-1), (-1,2,0)$ and $(1,2,3)$.

Q.3

(a) Derive the condition that the plane $lx + my + nz = p$ touches the cone $ax^2 + by^2 + cz^2 + 2fy + 2gzx + 2hxy = 0$. 7

(b) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 0$. 7

(P.T.O)

OR

Q.3

(*a*) Find the equation of the right circular cylinder whose axis is the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and whose radius is r.

(*b*) Find the equation to the lines in which the planes $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$

Q.4

(*a*) Find the intersection of the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and the central conicoid $ax^2 + by^2 + cz^2 = 1$. **7**

(*b*) Find the equations to the tangent planes to the conicoid $\frac{x^2}{16} + \frac{y^2}{9} - \frac{z^2}{4} = 1$ at the point (4,3,2). **7**

OR

(*a*) If P is a point on a central conicoid and if normal at P to the conicoid meets the principal planes at the points G_1, G_2, G_3 then show that $PG_1 : PG_2 : PG_3 :: a^{-1}: b^{-1}: c^{-1}$.

(*b*) Show that the plane $2x - 4y - z + 3 = 0$ touches the paraboloid $x^2 - 2y^2 = 3z$.

Q.5 Attempt any SEVEN in SHORT: **14**

(*a*) If α, β, γ are angle made by a line with positive directions of co-ordinate axis then find valuations of $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma$ and $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$.

(*b*) Give the equation of plane passing through the points $A(4,4,0), B(0,2,6), C(2,0,4)$.

(*c*) Find the angle between the planes $x + 2y - 4z = 12$ and $2x + 3y + 4z + 8 = 0$.

(*d*) Find the angle between the lines $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z-4}{-1}$ and $\frac{x-4}{1} = \frac{y+4}{2} = \frac{z+1}{2}$.

(*e*) Define orthogonal spheres.

(*f*) Write down all possible intersections of a Plane and a sphere.

(*g*) Find the center and radius of the sphere $x^2 + y^2 + z^2 + 4x + 12y - 8z = 6$.

(*h*) Define: Cone

(*i*) Define: Right Circular Cylinder.

(*j*) Give an equation of a Hyperboloid with two sheets.

(*k*) Give an equation of an Elliptic Paraboloid.

(*l*) How many normals can be drawn from a given point to a central conicoid and a paraboloid?

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Instructions: (i) All the questions are compulsory and carry equal marks.
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Q: 1 (*a*) If R is a ring with unity then prove the followings properties in a ring R:
(1) $(-1)\cdot a = -a, \ \forall a \in$ R.
(2) $(-1)\cdot(-1) = 1$. [7]

(*b*) If R is a ring satisfying $(a+b)^2 = a^2 + 2ab + b^2$ for all $a, b \in R$ then prove that the ring R is a commutative ring. [7]

OR

Q: 1 (*a*) Define an integral domain and prove that every field is an integral domain. Also Justify your answer whether the converse is true or false. [7]

(*b*) Define a Boolean ring and prove that a Boolean ring is a commutative ring. Also give an example of a Boolean ring. [7]

Q-2 (*a*) Define an ideal of a ring R.
Also prove that a nonempty subset I of a ring R is an ideal of R if and only if
(i) $a - b \in$ I , for all $a, b \in$ I and (ii) $a\cdot r$ and $r\cdot a \in$ I , for all $a \in$ I and for all $r \in$ R. [7]

(*b*) Show that the set $U = \{ \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} / \ a, b, c \in Z \}$ of triangular matrices is a subring of the ring $(M_2(Z), +, \bullet)$. [7]

OR

Q:2 (*a*) If R is a commutative ring with unity which has no proper ideal then prove that R is a field. [7]

(*b*) Define the Kernel K_ϕ of a Ring Homomorphism $\Phi : (R,+,\bullet) \rightarrow (R', \oplus, \odot)$ and prove that The homomorphism Φ is one-to-one if and only if $K_\phi = \{0\}$. [7]

Q: 3 (*a*) For nonzero polynomials $f, g \in D[x]$ Prove that $[f\cdot g] = [f] + [g]$. [7]

(*b*) Using Division algorithm for $f(x)$ and $g(x) \in Z_5[x]$ express $f(x)$ into the form $q(x)\, g(x) + r(x)$ for $f(x) = x^4 + 3x^2 + 2x + 4$ and $g(x) = x + 1 \in Z_5[x]$. [7]

OR

(P.T.O)

Q:3(*a*) Suppose $f(x) \in F[x]$ and $a \in F$.

Then prove that the reminder on dividing $f(x)$ by $x - a$ will be $f(a)$. [7]

(*b*) State the Eisenstein's criterion and prove that the polynomial $f(x) = x^n - p, n \geq 2$ is irreducible over Q for a prime p. [7]

Q: 4 (*a*) Define a maximal ideal and

Prove that $I = < p >$ is a maximal ideal for the ring Z of integers if and only if p is a prime. [7]

(*b*) Show that the polynomial $f(x) = x^3 + x^2 + 1$ is irreducible over Z_2 and $Z_2[x]/< f(x) >$ is a field with eight elements. [7]

OR

Q: 4 (*a*) Show that $Q[i] = \{a + bi / a, b \in Q\}$ is a subfield of the field C of complex numbers. [7]

(*b*) If F_1 and F_2 are subfields of a field F then prove that $F_1 \cap F_2$ also is a subfield of F. [7]

Q: 5 Attempt **any SEVEN** of the followings **in short:** [14]

(*a*) Define the terms: (i) A ring (ii) A commutative ring.
(*b*) Define an integral domain and a division Ring.
(*c*) Give an example of a division ring which is not a field.
(*d*) Give an example of a subring of a ring which is not an ideal of the ring.
(*e*) Give an example of a left ideal of a ring which is not an ideal of the ring.
(*f*) Define a polynomial in integral domain D and the degree of a nonzero polynomial in D.
(*g*) Define a primitive polynomial and a reducible polynomial.
(*h*) State the remainder theorem and the factor theorem for polynomials
(*i*) Define a monic polynomial and give one example of it.
(*j*) Define maximal and prime ideals of a commutative ring R with unity.
(*k*) Define a prime ideal and give an example of a prime ideal.
(*l*) Give an example of a finite field.

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