Seat No. : _______________

MF-111

April-2025

B.Sc., Sem.-V

MAT-305 : Mathematics

(Discrete Mathematics)

(Elective Course)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory and carry equal marks.

(2) Notations are usual everywhere.

(3) The right-hand side figures indicate marks of the question/sub-question.

1. (A) Prove that for all a, b $\in \mathbb{N}$, glb{a, b} = g.c.d.{a, b} and lub{a, b} = lcm{a, b} in the Poset ($\mathbb{N}$, D) with D as divides relation. **9**

1. (B) Draw the Hasse diagram of (S_{60}, D). **9**

OR

1. (A) Let (L, ≤) be a lattice then prove that for all a, b, c ∈ L, **9**

(i) $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$

(ii) $a * (b \oplus c) \geq (a * b) \oplus (a * c)$

1. (B) State and prove a non-empty subset S of a lattice (L, ∗, ⊕) is it's sublattice. **9**

2. (A) Prove that direct product of two lattices is a lattice. **9**

2. (B) Prove that (S_n, D) is distributive lattice. **9**

OR

2. (A) Prove that every distributive lattice is modular. Is the converse true ? Justify your answer. **9**

2. (B) Prove in a distributive lattice (L, ∗, ⊕) that **9**

$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a'' \oplus b = 1 \Leftrightarrow b' \leq a'$.

3. (A) Prove that direct product of two Boolean algebras is a Boolean algebra. **9**

3. (B) If f : (B, $*$, $\oplus$, ', 0, 1) $\rightarrow$ (P, $\wedge$, $\vee$, ~, α, β) is a Boolean homomorphism, then prove that (f(B), $\wedge$, $\vee$, ~, f(0), f(1)) is a Boolean algebra, where $f(B) = \{f(x) \mid x \in B\}$. **9**

OR

3. (A) State and prove Stone's representation theorem. **9**

3. (B) Prove that (S_n, D) is a Boolean algebra if and only if n is square free integer. **9**

4. Answer any **eight** of the following in short : **16**

(a) Define : Reverse relation and dual poset.

(b) Define : Chain and give an example of it.

(c) Draw a Hasse diagram of ($S_{25,}$ D).

(d) Define ring of subsets of a non-empty set X.

(e) State distributive laws in lattice.

(f) Define : Lattice as an algebraic structure.

(g) Define : Isomorphism of lattices.

(h) Write the De'Morgan's in lattice (L, $*$, $\oplus$, 0, 1).

(i) Define : Atom and Antiatom.

(j) Define : Minterm of n variables.

(k) Express $x_1 * x_2$ as sum of products canonical form over 3 variables x_1, x_2, x_3.

(l) Give an example of a Boolean Algebra.

Seat No. : _______________

MF-111

April-2025

B.Sc., Sem.-V

MAT-305 : Mathematics

(Number Theory)

(Elective Paper)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) Each question is compulsory.

(2) Figures to the right indicate marks of the question.

1. (A) State and prove the Division Algorithm theorem for integers. **9**

1. (B) Find the all positive solutions in the integers for the Diophantine equation $56x + 72y = 40$. **9**

OR

1. (A) Prove that the linear Diophantine equation $ax + by = c$ has a solution if and only if $\frac{d}{c}$, where d = g.c.d. (a, b).

Also prove that if x_0, y_0 is a solution of this equation then all other solutions are given by $x = x_0 + \left(\frac{b}{d}\right)^t$; $y = y_0 - \left(\frac{a}{d}\right)^t$, where t is any integer. **9**

1. (B) Using the Euclidean algorithm, find the g.c.d. for integer 1769 and 2378. Also obtain the integer x and y such that g.c.d. $(1769, 2378) = 1769x + 2378y$. **9**

2. (A) Define "Congruence modulo relation for a fixed positive integer n".

Also if $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$. **9**

2. (B) Using the Sieve of Eratosthenes; find all prime $p \leq 70$. **9**

OR

2. (A) State (only) Chinese remainder theorem. Also using this solve :

$2x \equiv 1 \pmod{3}$; $3x \equiv 1 \pmod{5}$; $5x \equiv 1 \pmod{7}$. **9**

2. (B) Prove that there are infinitely many primes of the form $4n + 3$. **9**

3. (A) State and prove Fermat's little theorem. **9**

3. (B) Solve the linear congruence $17x \equiv 9 \pmod{216}$. **9**

OR

3. (A) State and prove the Wilson's theorem. **9**

3. (B) In usual notation **9**

(i) Show that $2^{20} \equiv 1 \pmod{41}$ and

(ii) Find the remainder when the sum

$1! + 2! + 3! + \ldots\ldots\ldots + 50!$ is divisible by 12.

4. Attempt any **eight** in short : **16**

(1) Write down the canonical form for number 3668.

(2) Prove that the number N = 1571724 is divisible by 9 and 11.

(3) Find the l.c.m for two integers 657 and 306.

(4) Find $\phi(5040)$ and $\phi(360)$.

(5) Find the remainder when 5^{38} is divided by 11.

(6) What is the relation between $\phi(p)$; $\phi(q)$ and $\phi(pq)$? Where p and q are different prime.

(7) State (only) Euler's theorem.

(8) Define linear congruence equation.

(9) If $ax \equiv ay \pmod{n}$ and $(a, n) = 1$, then show that $x \equiv y \pmod{n}$.

(10) Define Euler's Phi-function.

(11) Define prime number and relatively prime number.

(12) If p is a prime number and $\frac{p}{ab}$, then show that $\frac{p}{a}$ or $\frac{p}{b}$.
