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2803N1152

Candidate's Seat No :________

B.Sc. Sem.-5 Examination

CC-303

Mathematics

March-2025

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks of the question/sub-question.
3) Notations used in this question paper carry their usual meaning.

Q-1 *a)* State and prove De Moivre's theorem. **(07)**

b) Find the four roots of the equation $z^4 - z^3 + z^2 - 1 = 0$. **(07)**

OR

Q-1 *a)* Suppose that $z_n = x_n + iy_n (n = 1, 2, 3, \ldots)$ and $z = x + iy$. Then prove that **(07)**

$$\lim_{n \to \infty} z_n = z \Leftrightarrow \lim_{n \to \infty} x_n = x \text{ and } \lim_{n \to \infty} y_n = y.$$

b) Define trigonometric and hyperbolic functions for the complex variables. Show that $|\sin z|^2 + |\cos z|^2 = \cosh 2y = ch\, 2y; z \in C$. Also, express $\sqrt{3} - i$ in the exponential form. **(07)**

Q-2 *a)* Prove that if a function $f(z) = u(x,y) + i\, v(x,y)$ is differentiable in D then u_x, u_y, v_x, v_y exist and $u_x = v_y$ and $u_y = -v_x$ in D. **(07)**

b) Define harmonic function. Find the Harmonic conjugate of $u(x,y) = e^x \sin y$, and corresponding analytic function in terms of z. **(07)**

OR

Q-2 *a)* Show that the function $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}; & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$ is not analytic at $z = 0$ $(z = (x, y))$; even if $f(z)$ satisfies Cauchy-Riemann equations at origin i.e. $z = 0$. **(07)**

b) Show that the function $w = \sin z$ satisfies the Cauchy-Riemann equations. Find its derivative. **(07)**

Q-3 *a)* Prove that if $f(z)$ is analytic at z_0 and $f'(z_0) \neq 0$, then $w = f(z)$ is conformal at z_0. **(07)**

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b) Find the image of the infinite strips (a) $\frac{1}{4} < y < \frac{1}{2}$ and (b) $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. Show them graphically. (07)

OR

Q-3 a) Find the bilinear transformations which map (07)

(i) $i, \ 1, \ \infty$ of z −plane onto $1, \ i, \ 0$ of w −plane respectively

(ii) $-1, \ 0, \ 1$ of z −plane onto $-i, \ 1, \ i$ of w −plane respectively.

b) Prove that the magnitude and the direction of angle between the lines $y = 2x$ and $y = x - 1$ remains same under the mapping $w = f(z) = z^2$. Sketch all curves and determine corresponding directions along them. (07)

Q-4 a) State and prove Euler's formula for the Fourier coefficients. (07)

b) Obtain Fourier series for the function $f(x) = x^2 - x$ in $[0, \ \pi]$. (07)

OR

Q-4 a) State and prove Bessel's inequality for the Fourier series. (07)

b) Find the half range sine series of $f(x) = \frac{\pi}{4} \cos x$ in $(0, \ \pi)$. (07)

Q-5 Answer the following questions in short (Any SEVEN) (14)

1 Simplify: $\left(\frac{1}{2-3i}\right)\left(\frac{1}{1+i}\right)$.

2 Prove that $|z| \le |Re\, z| + |Im\, z|$.

3 Find the modulus and principal argument of the complex number $1 - \sqrt{3}i$.

4 Is $u(x, \ y) = y^3 - 3x^2y$ harmonic?

5 Write the C-R equations and Laplace's equation in polar form.

6 Transform the curve $x^2 - y^2 = 4$ under the mapping $w = z^2$.

7 Find the angle of rotation produced by the transformation $w = z^2$ at point $z = 1 + i$.

8 Find the non-conformal points of the mapping $f(z) = 2z^3 + 15z^2 - 6z + 9$.

9 Find the singular points of $|z|^2$ and $\frac{1}{z}$.

10 Obtain $\int_{-\pi}^{\pi} \cos^2 nx \ dx$, for all $n \in N$.

11 Obtain $\int_{-\pi}^{\pi} \sin nx \ dx$, for all $n \in N$.

12 True / False:

(i) The Fourier series of the even function contains only constant or cosine terms.

(ii) The Fourier series of the odd function contains only constant or cosine terms.

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