Seat No. : _____

MC-103

March-2025

B.Sc., Sem.-V

MAT-302 : Mathematics

(Analysis – I)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.

(2) Figures to the right indicate full marks of the questions.

1. (A) State and prove Archimedean property. **7**

1. (B) For $x > -1$, show that $(1 + x)^n \geq 1 + nx$, $\forall n \in \mathbb{N}$. **7**

OR

1. (A) Prove that the interval (0, 1) is uncountable. **7**

1. (B) For $x, y \in \mathbb{R}$, prove that **7**

(i) $|x + y| \leq |x| + |y|$

(ii) $|x| \leq c$, then $-c \leq x \leq c$.

2. (A) State and prove Sandwich theorem. **7**

2. (B) Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is convergent. **7**

OR

2. (A) State and prove Bolzano-Weierstrass theorem. **7**

2. (B) Let $x_n = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+...+\frac{1}{n^2}$, show that the sequence $\{x_n\}$ is a Cauchy sequence. **7**

3. (A) Show that any polynomial of odd degree must have at least one real root. **7**

3. (B) Prove that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $(0, \infty)$ and uniformly continuous on (a, ∞), $a > 0$. **7**

OR

3. (A) Suppose that the function f is continuous on the interval [a, b] then prove that F is uniformly continuous on [a, b]. **7**

3. (B) Discuss uniform continuity of the function $f(x) = \sin\left(\frac{2\pi}{x}\right)$ on (0, 1). **7**

4. (A) State and prove Cauchy's mean value theorem. **7**

4. (B) Evaluate : **7**

(i) $\lim_{x\to 0} \frac{x - \sin x}{x^3}$

(ii) $\lim_{x\to 1} \frac{1}{\log x} - \frac{x}{x-1}$

OR

4. (A) State and prove Darbux's Theorem. **7**

4. (B) Verify Lagrange's mean value theorem for the function $f(x) = \sqrt{x} - x$ on [0, 4]. **7**

5. Attempt any **Seven** : (in Short) **14**

(1) If $A \subset B$, then $A \cap B =$ _____ and $A \cup B =$ _____.

(2) Find infimum and supremum of a set $\left\{1 - \frac{1}{n}\right\}$, $n \in \mathbb{N}$.

(3) If $f, g : \mathbb{R} \to \mathbb{R}$, $f(x) = 2x + 1$ and $g(x) = x^2 - 1$. Find $f \circ g$.

(4) Define Convergence of sequence.

(5) Give an example of sequence which is bounded and oscillatory.

(6) Find the cluster points of the sequence $\left\{\cos\left(\frac{n\pi}{3}\right)\right\}$.

(7) State Extreme value theorem.

(8) Show that the function $f(x) = \frac{x}{5}$ is uniformly continuous on R.

(9) Find the $\lim_{x \to 5} \left[\frac{x}{2}\right]$, if exists.

(10) State Roll's Theorem.

(11) Find first order derivative at $x = 0$ of $f(x) = \sin(e^{-3x})$.

(12) Evaluate $\lim_{x \to \infty} \frac{x-3}{x}$.
