Gujarat University
Semester Examination-March 2025
B. Sc. (Sem-V)
Subject :MATHEMATICS.
Course: MAT-301
(Linear Algebra-II)

Date : March 2025	**Time : 2 Hours 30 Minutes**	Max. Marks:70

Instruction : (i) All the questions are compulsory and carry equal marks.
(ii) Notations are usual everywhere.
(iii) The right hand side figures indicate marks of the question/sub-question.

Q-1 (*a*) If T : U→V is linear map , $v_0 \in$ R(T) and if T(u) = $\bar{0}_V$ has no solution then prove that the operator equation T(u) = v_0 has no solution. **[7]**

(*b*) Show that the 2×2 determinant function $f: R^2 \to R$ defined by $f(x,y) = x_1x_2 - y_1y_2$ for $x = (x_1, x_2)$, $y = (y_1, y_2) \in R^2$ is a bilinear form. **[7]**

OR

(*a*) State and Prove the Dual Basis Existence Theorem. **[7]**

(*b*) Find the dual basis of the basis B = {(1,0,0), (0,2,0), (0,1,3)} for the vector space V_3 . **[7]**

Q:2 (*a*) Prove that an orthogonal set of nonzero vectors is linearly independent. **[7]**

(*b*) If for $x = (x_1, x_2), y = (y_1, y_2) \in R^2$ the map < , > is defined as $< x, y > = x_1y_1 - x_1y_2 - x_2y_1 + 2x_2y_2$ then show that < , > is an inner product on R^2 . **[7]**

OR

(*a*) State and Prove the triangle inequality. **[7]**

(*b*) Apply the Gram-Schmidt orthogonalization process to the basis B = {(1, 2) (4, 1) } in order to get the orthonormal basis for V_2 . **[7]**

Q:3 (*a*) If i ≠ j , α∈R and if *det* : $V^n \to R$ is a function satisfying the expected properties of the determinant then prove the followings :

(i) $det(v_1, v_2, \dots, v_i, \dots, v_j, \dots, v_n) = det(v_1, v_2, \dots, v_i + \alpha v_j, \dots, v_j, \dots, v_n)$

(ii) $det(v_1, v_2, \dots, v_i, \dots, v_j, \dots, v_n) = -det(v_1, v_2, \dots, v_j, \dots, v_i, \dots, v_n)$. **[7]**

(*b*) If $A=\begin{bmatrix}1 & 2 & 3 & 4\\ 5 & 0 & 6 & 7\\ 0 & 8 & 9 & 10\\ 0 & 0 & 0 & 1\end{bmatrix}$ then compute $detA$ without expansion. **[7]**

OR

(*a*) Derive the formula of finding area of a parallelogram in terms of a 2×2 determinant. **[7]**

(*b*) Use the Cramer's rule to solve :

$$x - 2y + z = 4$$
$$x + y - z = 4$$
$$x - 2y + z = 6.$$

[7]

Q:4 (*a*) Express the characteristic equation of 2×2 matrix in terms of its trace and determinant. Also prove that a 2×2 real and symmetric matrix has only real eigen values. **[7]**

(*b*) Diagonalize the matrix $A = \begin{bmatrix}2 & 0 & 1\\ 0 & 3 & 0\\ 1 & 0 & 2\end{bmatrix}$. **[7]**

OR

(*a*) State and Prove the Caley-Hamilton's Theorem. **[7]**

(*b*) Identify the quadric in R^3 given by $f(x, y, z) \equiv 4xz + 4y^2 + 8y + 8 = 0$. **[7]**

Q-5 Attempt any **SEVEN** of the followings in **Short** : **[14]**

(*a*) Define a linear functional and the Dual Space of a vector space.

(*b*) Define homogeneous and nonhomogeneous operator equations.

(*c*) Define a Bilinear form and an Annihilator.

(*d*) Define a Euclidian Space and a Unitary space.

(*e*) Define orthogonal projection of a vector along a nonzero vector.

(*f*) Define an orthogonal linear map and orthogonal complement of a subspace of ips V.

(*g*) State any two expected properties of the determinant function.

(*h*) State the Laplace expansion for finding the value of a determinant.

(*i*) Find detA without expansion if $A = \begin{bmatrix}1 & 0 & 0\\ 2 & 3 & 0\\ 4 & 5 & 6\end{bmatrix}$

(*j*) Define an eigen value and eigen vector of an endomorphism.

(*k*) Define a symmetric linear map and a quadric.

(*l*) State the spectral theorem.

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