

## M.Sc. Sem.-4 Examination

510

## Mathematics (EA)

April-2025

[Max. Marks : 70]

Time : 2-30 Hours]

**Instructions:** All questions are compulsory. Use of non-programmable scientific calculator is allowed.

- Q.1 (a) Derive basic EOQ model. (07)
- (b) Find the most economic batch quantity of a product on a machine if the production rate of that item on the machine is 200 pieces per day and the demand is uniform at the rate of 100 pieces per day. The ordering cost is Rs. 200 per batch and the cost of holding one item in inventory is Rs. 0.81 per day. How will the batch quantity vary if the production rate is infinite? Also find cycle time and length of production cycle. (07)

OR

- Q.1 (a) Derive EOQ for order – level, lot – size system. (07)
- (b) Three machines are produced by a factory in lots. A factory has floor space of 600 square feet. The demand rate, purchase cost, ordering cost and floor space required are given in the following table: (07)

Item	1	2	3
Demand rate (unit/year)	5,000	2,000	10,000
Purchase cost per unit (Rs.)	20	15	10
Ordering cost per order (Rs.)	100	150	200
Floor space required (sq. ft)	0.60	0.75	0.30

The factory uses an inventory carrying charge at 20% of average inventory per year. If no shortages are allowed, determine the optimal lot-size for each item under given floor constraints.

- Q.2 (a) Derive only probability function of  $m/m/c$  queuing model (Infinite capacity). (07)
- (b) A departmental store has a single cashier. During the rush hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. Assume that the conditions for the use of the single channel queuing model apply. (07)
- What is the probability that the cashier is idle?
  - What is the average number of customers in the queuing system?
  - What is the average time a customer spends in the system?
  - What is the average number of customers in the queue?
  - What is the average time a customer spends in the queue waiting for service?

OR

- Q.2 (a) Explain pure birth process. (07)
- (b) In a maintenance shop, the inter-arrival times at tool crib are exponential with an average time of 10 minutes. The length of the service (i.e. the amount of time taken by the tool crib operator to meet the needs of the maintenance man) time is assumed to be exponentially distributed with a mean 6 minutes. Find (07)
- The probability that a person arriving at the booth will have to wait.
  - The average length of the queue that forms.
  - The average time that an operator spends in the queuing system.

- d) The estimate of the fraction of the day that the tool crib operator will be idle.
- e) The probability that there will be six or more operators waiting for the service.

- Q.3 (a) State and prove necessary and sufficient condition for the existence of the saddle point. (07)
- (b) Reduce the following game as far as possible and then solve it. (07)

<b>Player A</b>	<b>Player B</b>				
		<b>B<sub>1</sub></b>	<b>B<sub>2</sub></b>	<b>B<sub>3</sub></b>	<b>B<sub>4</sub></b>
	<b>A<sub>1</sub></b>	-1	2	3	0
	<b>A<sub>2</sub></b>	-4	-1	-1	0
	<b>A<sub>3</sub></b>	-1	1	1	-4
<b>A<sub>4</sub></b>	4	-1	2	-7	

OR

- Q.3 (a) Solve the following game by using simplex algorithm. (07)

<b>Player A</b>	<b>Player B</b>			
		<b>B<sub>1</sub></b>	<b>B<sub>2</sub></b>	<b>B<sub>3</sub></b>
	<b>A<sub>1</sub></b>	2	-2	3
	<b>A<sub>2</sub></b>	-3	5	-1

- (b) Solve the following game by using graphical method. (07)

<b>Player A</b>	<b>Player B</b>		
		<b>I</b>	<b>II</b>
	<b>I</b>	1	3
	<b>II</b>	3	1
	<b>III</b>	5	-1
<b>IV</b>	6	-6	

- Q.4 (a) A manufacturer of goods has only three options available for the existing plant: expand the present plant, build a new plant and subcontract out extra production to other manufacturers. The future events concern demand for the product. They are: High demand, Moderate demand, Low demand and failure. The payoff table is given below: (Price in rupees) (07)

<b>Decision alternatives</b>	<b>State of nature</b>			
	<b>High</b>	<b>Moderate</b>	<b>Low</b>	<b>Failure</b>
<b>Expand</b>	5,00,000	2,50,000	-2,50,000	-4,50,000
<b>Build</b>	7,00,000	3,00,000	-4,00,000	-8,00,000
<b>Subcontract</b>	3,00,000	1,50,000	-10,000	-1,00,000

Which strategy should the concerned executive choose on the basis of -

- (a) Maximax Criterion
  - (b) Maximin Criterion
  - (c) Hurwicz criterion (degree of optimism = 0.6)
  - (d) Laplace criterion
- (b) A company is considering whether to launch a new product or not. The success of the idea depends on the ability of a competitor to bring out a competing product (estimated at 60%) and the relationship of the competitor's price to the firm's price. (07)

The following table shows the profits for each price range that could be set by the company related to the possible competing prices.

If the company's price is	If the competitor's price is			If no competitor
	Low	Medium	High	
Low	30	42	45	50
Medium	34	45	49	70
High	10	30	53	90

The company must set its price first because its product will be on the market earlier so that the competitor will be able to react to the price. Estimates of the probability of a competitor's price are shown in the following table.

If the company's price is	Competitor's price expected to be		
	Low	Medium	High
Low	0.8	0.15	0.05
Medium	0.2	0.7	0.1
High	0.05	0.35	0.6

- Draw a decision tree and analyze the problem.
- Recommend what the company should do.

**OR**

- Q.4 (a)** Farmer Mahesh can plant either corn or soybeans. The probabilities that the next harvest prices will go up, stay the same, or go down are 0.25, 0.30, and 0.45, respectively. The payoff table is given below. (Price in rupees) (07)

Decision alternative	State of nature		
	Prices go up	Prices unchanged	Prices go down
Corn	30,000	0	-35,000
Soyabean	10,000	0	-5000

- Draw the decision tree of given problem.
  - Which crop should Mahesh plant?
- (b)** ABC industries must decide to build a large or a small plant to produce a product which is expected to have a market life of 10 years. A large plant will cost 28,00,000 to build and put into operation, while a small plant will only cost 14,00,000 to build and put into operation. The company's best estimate of a discrete distribution of sales over the 10 year period is (07)

High demand: Probability = 0.5

Moderate demand: Probability = 0.3

Low demand: Probability = 0.2

The annual conditional outcomes under the various combinations of plant sizes and market sizes are as follows

		Demand		
		High	Moderate	Low
Plant	Large	10,00,000	6,00,000	-2,00,000
	Small	2,50,000	4,50,000	5,50,000

- a) Draw the decision tree of given problem.
- b) Find the expected value of the given problem.

**Q.5** Attempt any **SEVEN** out of **TWELVE**:

**(14)**

- (1) Define: Holding cost and Shortage cost
- (2) Describe total inventory cost.
- (3) Obtain the EOQ using given information.  
Annual demand = 20,000 units, ordering cost = Rs. 150 per order, and inventory carrying cost is 24% of average inventory value.
- (4) Explain balking and jockeying event.
- (5) Define: Transient state
- (6) The arrival rate of a customer at a service window of cinema hall is 10 per minute and service rate is 4 per minute. If service station has three servers, then find the utilization rate of system.
- (7) Explain: Pure strategy and Mix strategy
- (8) Define: saddle point
- (9) What is two-person zero sum game?
- (10) Explain Expected Value of Sample Information (EVSI).
- (11) Farmer McCoy can plant either corn or soybeans. The probability that the next harvest prices will go up, stay the same, or go down are 0.25, 0.30, and 0.45, respectively. If the prices go up, the corn crop will net \$30,000 and the soybeans will net \$10,000. If the prices remain unchanged, McCoy will (barely) break even. But if the prices go down, the corn and soybeans crops will sustain losses of \$35,000 and \$5000, respectively. Represent McCoy's problem as a decision tree.
- (12) Draw the graph of economic analysis of inventory models.

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