Seat No. : _____

AF-128

April-2025

B.Sc., Sem.-VI

CC-310 : Mathematics

(Graph Theory)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (i) There are **FIVE** questions in this paper. **All** questions are compulsory.
(ii) Figures to the right indicate full marks of the question/sub-question.
(iii) Notations are usual everywhere.

1. (a) Prove that the complete graph K_n has $\frac{n(n-1)}{2}$ edges. **7**

(b) Define isomorphism of graphs. Show that the following graphs(Fig-l) are isomorphic. **7**

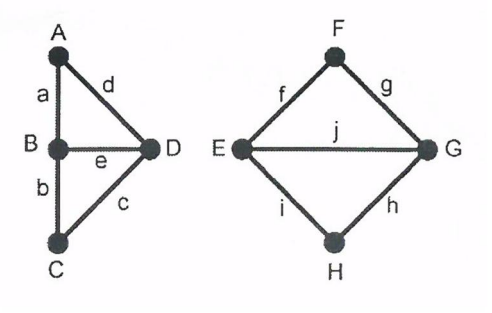

Fig – 1

OR

1. (a) Given any two vertices u and v of a graph G, prove that every u – v walk contains a u – v path. **7**

(b) Let G be the following graph (Fig-2). **7**

(i) Find a closed walk of length 6. Is your walk a trail ?

(ii) Find an open walk of length 12. Is your walk a path ?

(iii) Find a closed trail of length 6. Is your trail a cycle ?

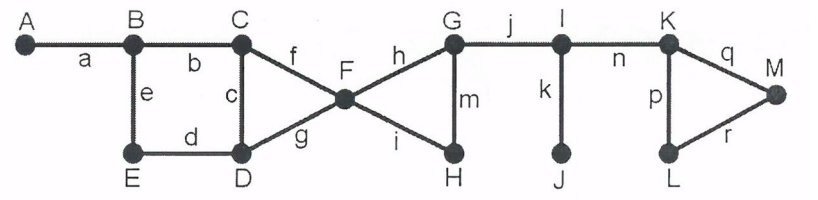

Fig. – 2

2. (a) If G is an acyclic graph with n vertices and k connected components then prove that it has n – k edges. **7**

(b) Without drawing the actual graph, determine whether the graph G is connected or not, whose adjacency matrix is $A(G) = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$. **7**

OR

2. (a) Let T be a tree with atleast two vertices and let $P = u_0u_1 \ldots u_n$ be a longest path in T (so that there is no path in T of length greater than n). Then prove that both u_0 and u_n have degree 1. **7**

(b) Write down the adjacency and incidence matrices of the following graphs (Fig-3). **7**

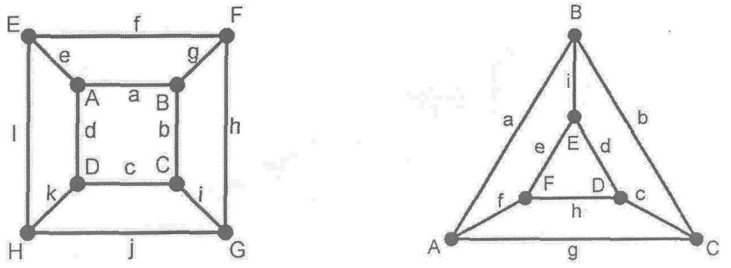

Fig – 3

3. (a) Prove that a graph G is connected if and only if it has a spanning tree. **7**

(b) Find Connectivity k(G) for the following graphs. If k(G) = 1 identify the cut vertices. (Fig-4) **7**

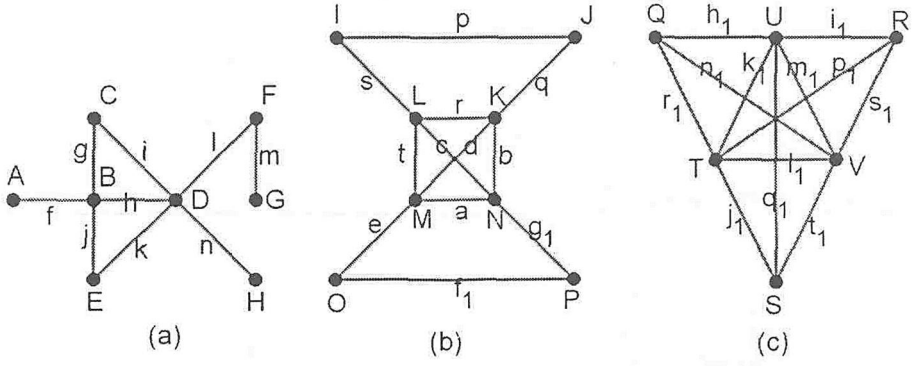

Fig – 4

OR

3. (a) Let G be a simple graph with at least three vertices. If for each pair of distinct vertices u and v of G, there are two internally disjoint u – v paths in G, then prove that G is 2 – connected. **7**

(b) Give a list of all spanning trees, including isomorphic ones, of the complete graph K_4. **7**

4. (a) Write a short note on Konigsberg seven bridges problem. **7**

(b) Define closure of a graph. Prove that a simple graph G is a Hamiltonian if and only if its closure c(G) is Hamiltonian. **7**

OR

4. (a) Prove that a connected graph G is Euler if and only if the degree of every vertex is even. **7**

(b) Find closure of the graph (Fig-5) : **7**

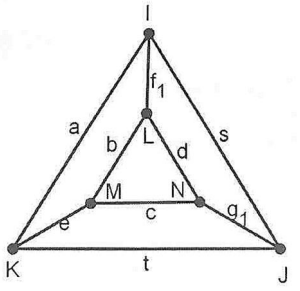

Fig – 5

5. Attempt any **SEVEN** of the followings in short : **14**

(i) Is every trail a path ? If no, give an example.

(ii) Give an example of a 4-regular graph with 2 vertices.

(iii) Define complete bipartite graph and give an example.

(iv) Find the graph G - {L, M, N} for the graph in Fig-5.

(v) If connected graph G has 23 vertices, what is the minimum possible number of edges in G ? Why ?

(vi) List all bridges in the graph in Fig-2.

(vii) Define n-connected graph. Is the graph in Fig-5 2-connected ?

(viii) Define minimal spanning tree.

(ix) Draw a spanning tree (if exists) of the graph in Fig-2.

(x) Find an Euler tour in the complete graph K_4 (if exists).

(xi) Define Hamiltonian Cycle. Is the graph in Fig-5 Hamiltonian ?

(xii) Define a maximal non-Hamiltonian graph and give an example.