2/16

2703N1126

Candidate's Seat No :________

M.Sc. Sem.-3 Examination

503

Mathematics EA Adv.

March-2025

Time : 2-30 Hours] **[Max. Marks : 70**

1. (A) Let $f(x,y,z) = x^2e^{x-y+3z}$. Compute the differential df. Use the differential to estimate the difference $f(1.1, 1.2, -0.1) - f(1,1,0)$. **7**

   (B) Show that $f(x,y) = (x^2+2y^2)e^{-x^2-y^2}$ has an absolute minimum and maximum on $\mathbb{R}^2$, and find them. **7**

OR

   (A) Find the 3^{rd}- order Taylor polynomial of $f(x,y) = x + \cos \pi y + x \log y$ based at $\bar{a} = (3,1)$ **7**

   (B) Find and classify the critical points of $f(x,y) = x^2 + 3y^4 + 4y^3 - 12y^2$. **7**

2. (A) Let $(u,v) = f(x,y) = (x-2y, 2x-y)$. Then compute the inverse transformation $(x,y) = f^{-1}(u,v)$. Also find the image in the uv- plane of the triangle bounded by the lines $y = x, y = -x$, and $y = 1-2x$. **7**

   (B) Find an equation for the tangent plane to the parametrized surface $x = \frac{1}{u+v},\ y = -(u+e^v),\ z = u^3$ at the point $(1,\ -2,\ 1)$ **7**

OR

   (A) Find a parametric description of the line obtained by the intersection of the planes $x - 2y + z = 3$ and $2x - y - z = -1$. **7**

   (B) Let $(u,v) = \mathbf{f}(x,y) = (x^2+2xy+y^2, 2x+2y)$. Compute the Jacobian $\det D\mathbf{f}$. Draw a sketch of the images of some of the lines x = constant and y = constant in the uv–plane. Find a formula for a local inverse of $\mathbf{f}$. **7**

3. (A) Find the centroid of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. **7**

   (B) An object with mass density $\rho(x,y,z) = yz$ occupies the cube $\{(x,y,z) : 0 \le x,y,z \le 2\}$. Find its mass and center of mass. **7**

(P.T.O)

N1126-2

OR

(A) Find the centroid of the half-cone $\sqrt{x^2+y^2} \leq z \leq 1,\ x \geq 0$. **7**

(B) Find the volume of the region inside both the sphere $x^2+y^2+z^2 = 4$ and the cylinder $x^2+y^2 = 1$. **7**

4. (A) Compute $\iint_S \overline{F} \cdot \overline{n}\, dA$, where $\overline{F}(x,y,z) = x^2\hat{i} + z\hat{j} - y\hat{k}$ and S is the unit sphere $x^2+y^2+z^2 = 1$, oriented so that the normal points outward(away from the center). **7**

(B) Compute $\int_C (xe^{-y}\, dx + \sin \pi x\, dy)$, where C is the portion of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$. **7**

OR

(A) Find the centroid of the curve $y = \cosh x, -1 \leq x \leq 1$. **7**

(B) Compute $\iint\limits_C (x^2+y^2)dA$, where S is the portion of the sphere $x^2+y^2+z^2 = 4$ with $z \geq 1$. **7**

5. **Attempt any seven of the following.** **14**

(1) Find the directional derivative of the function $f(x,y) = xy^2 + \cos \pi xy$ at the point $(\frac{1}{2}, -1)$ in the direction $(\frac{3}{5}, \frac{4}{5})$.

(2) Find the tangent plane to the surface $z = x^2 - y^3$ at the point $(3,-1,10)$.

(3) Find critical points of the function $f(x,y) = x^2 + 3y^4 + 4y^3 - 12y^2$.

(4) Compute $Df(0,0)$ for the function $f(u,v) = (u^2 - 5v, ve^{2u}, 2u - log(1+v^2))$.

(5) Let $F = (f,g,h)$, where $f(x,y,z) = x+y-z$, $g(x,y,z) = x-y+z$, and $h(x,y,z) = x^2+y^2+z^2-2yz$, then find rank of DF.

(6) Evaluate $\int_0^2 \int_{\frac{y}{2}}^1 (xy+y^2)dxdy$.

(7) Evaluate the double integral $\iint\limits_R y dA$, where $R = [0,2] \times [0,1]$.

(8) Find the area of the part of the surface $z = xy$ inside the cylinder $x^2+y^2 = 9$.

(9) Find arc length of the curve $f(t) = (a\cos t, a\sin t, bt), t \in [0, 2\pi]$.

(10) State Green's theorem.

(11) State the Divergence theorem.

(12) Compute the divergence of the vector field $\mathbf{F}(x,y,z) = (xy^2,\ xy,\ xy)$.