3/11

2603N1104

Candidate's Seat No :________

M.Sc. Sem.-3 Examination

502

Mathematics

March-2025

Time : 2-30 Hours] **[Max. Marks : 70**

1. (A) Describe the symmetries of an equilateral triangle. Construct the corresponding Cayley table. **7**

   (B) Let H and K be subgroups of an Abelian group G. Show that the set $HK = \{hk \mid h \in H, k \in K\}$ is a subgroup of G. **7**

OR

   (A) Show that every subgroup of a cyclic group is cyclic. List all subgroups of $\mathbb{Z}_{24}$. **7**

   (B) Determine the subgroup lattice for the group Z_{36}. **7**

2. (A) State and prove Cayley's theorem. **7**

   (B) Determine the number of elements in S_7 of order 2, 3, and 12. **7**

OR

   (A) State and prove Lagrange's theorem. **7**

   (B) Determine the number of cyclic subgroups of order 10 in $\mathbb{Z}_{100} \oplus \mathbb{Z}_{25}$. **7**

3. (A) Let G be a group and let $Z(G)$ be the center of G. If $G/Z(G)$ is cyclic, prove that G is Abelian. **7**

   (B) If N and M are normal subgroups of a group G, prove that NM is also a normal subgroup of G. **7**

OR

   (A) If a group G is the internal direct product of a finite number of subgroups $H_1, H_2, \ldots, H_n$, prove that G is isomorphic to the external direct product of $H_1, H_2, \ldots, H_n$. **7**

(P.T.O)

(B) State and prove first isomorphism theorem. **7**

4. (A) Define conjugacy class of an element a in a group G. Calculate all conjugacy classes for the dihedral group D_4 and verify the class equation. **7**

(B) If G is a group of order pq, where p and q are primes, $p < q$, and p does not divide $q-1$, prove that G is cyclic. Is there any group G, such that $|G/Z(G)| = 77$? Explain. **7**

OR

(A) State and prove Index theorem. Is there any simple group of order 80? Explain. **7**

(B) Prove that a Sylow p-subgroup of a finite group G is a normal subgroup of G if and only if it is the only Sylow p-subgroup of G. **7**

5. **Attempt any seven of the following.** **14**

(1) The order of the center of the dihedral group D_{29} is

(A) 1 (B) 2 (C) 3 (D) 4

(2) Which of the following groups are cyclic?

(A) The group of integers $\mathbb{Z}$

(B) The group $\mathbb{Z} \oplus \mathbb{Z}$

(C) Dihedral group D_4

(D) The group $U(10)$ under multiplication modulo 10.

(3) Let a and b be elements of a group. If $|a| = 10$ and $|b| = 21$, then $|\langle a\rangle \cap \langle b\rangle|$ equals

(A) 10 (B) 21 (C) 5 (D) 1

(4) In $\mathbb{Z}_{18}$, the order of the element 15 is

(A) 6 (B) 3 (C) 5 (D) 2

(5) The order of the permutation $(1\ 2)(1\ 3\ 4)(1\ 5\ 2\ 6)$ is

(A) 24 (C) 8

(B) 4 (D) 12

(6) Let α be an odd permutation of the symmetric group S_7. Then the possible disjoint cycle structures of α are

(A) $(\underline{4})(\underline{1})(\underline{1})(\underline{1})$

(B) $(\underline{4})(\underline{3})$

(C) $(\underline{3})(\underline{3})(\underline{1})$

(D) $(\underline{2})(\underline{2})(\underline{2})(\underline{1})$

(7) The number of automorphisms of the group $\mathbb{Z}_{15}$ is

(A) 1 (B) 8 (C) 5 (D) 15

(8) Let ϕ be an automorphism of dihedral group D_6. Then $\phi(R_{180}) =$ ________.

(A) R_0 (B) R_{120} (C) R_{60} (D) R_{180}

(9) What is the order of the factor group $\mathbb{Z}_{30}/\langle 5 \rangle$?

(A) 5 (B) 6 (C) 4 (D) 3

(10) In the symmetric group S_3, the conjugacy class of $(1\ 2)$, $cl((1\ 2))$ equals

(A) $\{(1\ 2), (1\ 3)\}$

(B) $\{(1\ 2), (1\ 2\ 3), (1\ 3\ 2)\}$

(C) $\{(1\ 2), (1\ 3), (2\ 3)\}$

(D) $\{(1\ 2), (2\ 3)\}$

(11) How many Abelian groups (up to isomorphism) are there of order 72?

(A) 2 (B) 4 (C) 6 (D) 8

(12) Which of the following groups are simple?

(A) S_3 (B) $\mathbb{Z}_{29}$ (C) $\mathbb{Z}_{12}$ (D) A_5
