Seat No. : _______________

FD-141

February-2025

M.Sc., Sem.-I

MAT-404 : Mathematics

(Ordinary Differential Equations)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) Find the general solution of $y'' - 3y' + 2y = 14 \sin 2x - 18 \cos 2x$. **7**

1. (B) Solve $y'' - y' + xy = 0$ in terms of power series in x. **7**

OR

1. (A) Find a particular solution of $y'' + y = \sec x \tan x$. **7**

1. (B) Solve $y'' + (1 + x)y' - y = 0$ in terms of power series in x. **7**

2. (A) Find two independent Frobenius series solutions of $xy'' - y' + 4x^3y = 0$. **7**

2. (B) Find the general solution of $(x^2 - x - 6)y'' + (5 + 3x)y' + y = 0$ near the singular point $x = 3$. **7**

OR

2. (A) Find the general solution of the Gauss hypergeometric equation.
$x(1 - x)y'' + [c - (a + b + 1)x]y' - aby = 0$ near the singular point $x = 0$. **7**

2. (B) Determine the nature of the point $x = \infty$ for the Legendre equation
$(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$. **7**

3. (A) State the Rodrigues' formula for Hermite polynomials. (Do not prove)
Calculate Hermite polynomials $H_n(x)$, for n = 0, 1, 2, 3, 4. **7**

3. (B) Obtain the recursion formula $T_n(x) + T_{n-2}(x) = 2xT_{n-1}(x)$ for the Chebyshev polynomials $T_n(x)$. Calculate Chebyshev polynomials $T_2(x)$, $T_3(x)$ and $T_4(x)$ by taking $T_0(x) = 1$ and $T_1(x) = x$. **7**

OR

3. (A) Obtain the recursion formula for the Legendre polynomials $P_n(x)$. Assume that $P_0(x) = 1$ and $P_1(x) = x$, calculate $P_2(x)$, $P_3(x)$ and $P_4(x)$. **7**

3. (B) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$, if $m \neq n$. **7**

4. (A) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. **7**

4. (B) Prove that $\int_0^1 x J_p(\lambda_m x) J_p(\lambda_n x) dx = 0$, if $m \neq n$. **7**

OR

4. (A) Consider the initial value problem. **7**

$y' = 2x(1 + y)$, $y(0) = 0$

Find successive approximations $y_0(x)$, $y_1(x)$, $y_2(x)$, $y_3(x)$ using Picard's method.

4. (B) Derive addition formula for Bessel function. **7**

5. Attempt any **seven** of the following : **14**

(1) Find the general solution of $y'' + y' - 2y = 0$.

(2) The equation $(1 - x^2)y'' - 2xy' + 2y = 0$ has the solution $y_1 = x$. Find another linearly independent solution y_2.

(3) Find the radius of convergence of $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$.

(4) Is $x = 0$ an ordinary point for $xy'' + (\cos x)y = 0$? Justify your answer.

(5) Determine the nature of the point $x = 1$ for $x^3(x - 1)y'' - 2(x - 1)y' + 3xy = 0$.

(6) Find the indicial equation and its roots for $2xy'' + (3 - x)y' - y = 0$.

(7) Show that $T_n(x) = \frac{1}{2}\left[(x + \sqrt{x^2 - 1})^n + (x - \sqrt{x^2 - 1})^n\right]$.

(8) State the Rodrigues' formula for Legendre polynomial $P_n(x)$.

(9) What is the value of the integral $\int_{-1}^{1} P_5(x) P_5(x) dx$?

(10) Show that between any two positive zeros of $J_0(x)$, there is a zero of $J_1(x)$.

(11) Write down the infinite continued fraction expansion of the ratio $J_{p-1}(x)/J_p(x)$.

(12) Does the function $f(x, y) = y^{1/2}$ satisfy the Lipschitz condition on the rectangle $|x| \leq 1$ and $0 \leq y \leq 1$? Justify your answer.
