0502E1469

Candidate's Seat No :________

MSc Sem.-1 Examination

404

AMS

Time : 2-30 Hours] **February-2025** **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Let $A = \begin{pmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{pmatrix}$. Analyze the long-term behavior of the dynamical system defined by $x_{k+1} = Ax_k$ ($k = 0,1,2,\ldots$), with $x_0 = \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$ **(07)**

(b) A small remote village receives radio broadcasts from two radio stations, a news station, and a music station. Of the listeners who are tuned to the news station, 70% will remain listening to the news after the station break that occurs each half hour, while 30% will switch to the music station at the station break. Of the listeners who are tuned to the music station, 60% will switch to the news station at the station break, while 40% will remain listening to the music. Suppose everyone is listening to the news at 8:15 A.M. **(07)**

a. Give the Stochastic matrix that describes how the radio listeners tend to change stations at each station break. Label the rows and columns.
b. Give the initial state vector.
c. What percentage of the listeners will be listening to the music station at 9:25 A.M. (after the station breaks at 8:30 and 9:00 A.M.)?

OR

(a) In a certain town, 30 percent of the married women get divorced each year and 20 percent of the single women get married each year. There are 8000 married women and 2000 single women, and the total population remains constant. Investigate the long-range prospects if these percentages of marriages and divorces continue indefinitely into the future. **(07)**

(b) Find all the possible Jordan canonical forms of the matrix A whose order is 6×6 defined over R (set of all real numbers) with characteristic polynomial $(x-3)^2(x-2)^4$ and minimal polynomial $(x-3)(x-2)^2$. **(07)**

Q.2 **(a)** Given matrix $A = \begin{pmatrix} 0 & 2 & -1 \\ 2 & 3 & 2 \\ -1 & -2 & 0 \end{pmatrix}$, find an orthogonal matrix U that diagonalizes the matrix A. **(07)**

(b) Let R^3 have the inner product $\langle (x_1, x_2, x_3), (y_1, y_2, y_3) \rangle = x_1y_1 + 2x_2y_2 + 3x_3y_3$. Use the Gram-Schmidt process to transform the basis vectors $u_1 = (1,1,1), u_2 = (1,1,0), u_3 = (1,0,0)$ into an orthonormal basis. **(07)**

OR

(P.T.O)

(a) Given the quadratic equation $5x^2 - 4xy + 8y^2 - 36 = 0$ find a change of coordinates so that the resulting equation represents a conic in standard position. Sketch the rough figure of the required conic. **(07)**

(b) Determine the Algebraic Multiplicity and Geometric Multiplicity of each eigenvalue of the following matrix: **(07)**

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

Q.3 **(a)** Factorize $A = \begin{pmatrix} 2 & -2 & 4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{pmatrix}$ by Crout's method. **(07)**

(b) Apply Gauss-Seidel method to solve the system of equations **(07)**

$$20x + y - 2z = 17; \quad 3x + 20y - z = -18; \quad 2x - 3y + 20z = 25$$

(Hint: Perform first three iterations which easily verified the exact solution)

OR

(a) Determine $\|.\|_F$, $\|.\|_\infty$, and $\|.\|_1$ for each of the following matrices: **(07)**

i. $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

ii. $\begin{pmatrix} 5 & 0 & 5 \\ 4 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$

(b) Find the Singular Value Decomposition (SVD) of the matrix: **(07)**

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

Also, find the closest (with respect to Frobenius norm) matrix of rank 1.

Q.4 **(a)** Prove that $\mathbb{N}$ (a set of nature numbers) with respect to divisibility relation forms a lattice. **(07)**

(b) **(07)**
- (i) Give an example of a relation which is neither reflexive nor irreflexive.
- (ii) Define partially ordered set.
- (iii) Define chain.
- (iv) Consider X = {a, b, c}. Is R = {(a, b)} an antisymmetric relation? Justify your answer.
- (v) Is S_{90} w.r.t divisibility relation form a Boolean algebra? Justify your answer.

OR

(a) Prove that as a Boolean algebra S_{42} and $P(X),\ X = \{a, b, c\}$ are isomorphic. **(07)**

(b) Minimize the following Boolean expression using K-map **(07)**

$$f(A, B, C, D) = \sum m(0,1,5,7,8,10,14,15)$$

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) State (only) Diagonalization of the matrices with suitable example.

(2) Express the following quadratic forms in matrix notation:

$$2x^2 + 3y^2 - 5z^2 - 2xy + 6xz - 10yz$$

(3) Check whether the following matrices are similar or not.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(4) State the necessary condition for Gauss-Seidel iterative method with suitable example.

(5) Identify the curve of the quadratic equation: $x^2 + 4xy + y^2 + 4x + 6y + 4 = 0$.

(6) Determine $\|.\|_F$ *and* $\|.\|_\infty$ for each of the following matrices: $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

(7) State (only) Orthonormal set with suitable example.

(8) State (only) the difference between the algorithm of Crout's and Doolittle method.

(9) Determine norm $\|A\|_1$, where $A = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$.

(10) Consider $X = \{1,2,3\}$ and power set of X as $P(X)$ with respect to set inclusion relation. Does a set inclusion relation on $P(X)$ forms a partially ordered relation? Justify your answer.

(11) Draw Hasse diagram of (S_{45}, D).

(12) Using Boolean Algebra, prove that: $(A + B)(A + C) = A + BC$

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