0402E1401

Candidate's Seat No :________

MSc Sem.-1 Examination

403

Statistics

Time : 2-30 Hours] **February-2025** **[Max. Marks : 70**

Note: Attempt all questions.

Q. 1

(i) State and prove Neyman's factorization theorem for discrete case. [7]

(ii) State and prove Basu's theorem. [7]

OR

(i) State and prove Lehmann - Scheffe theorem. [7]

(ii) Let $x_1, x_2, \cdots, x_n$ be a random sample from N(μ,σ^2), where μ &σ^2 are unknown. Obtain minimal sufficient statistic for μ & σ^2 . [7]

Q.2

(i) State and prove Rao-Blackwell theorem. [7]

(ii) Let $X_1, X_2, \cdots, X_n$ be a random sample from $U[0,\theta]$ population. Obtain minimum variance unbiased estimator for θ. [7]

OR

(i) Define minimum variance unbiased estimator and minimum variance bound unbiased estimator and explain clearly the difference between them. Prove that minimum variance unbiased estimator is essentially unique. [7]

(ii) Let $X_1, X_2, \cdots, X_n$ be a random sample from a rectangular distribution with p.d.f. $f(x;\theta) = 1/\theta$, $0 \le x \le \theta$. Find MVU estimators of θ and $3\theta + 5$. [7]

Q.3

(i) Define MLE. State its properties and prove any one of them. [7]

(ii) Obtain 100(1-α)% confidence limits (for large samples) for the parameter λ of the Poisson distribution [7]

$$f(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!};\ x = 0, 1, 2, \cdots$$

OR

(i) Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size n. Also find its variance. [7]

(ii) In random sampling from normal population $N(\mu,\sigma^2)$, find the maximum likelihood estimators for (a) μ when σ^2 is known, (b) σ^2 when μ is known, and (c) the simultaneous estimation of μ and σ^2. [7]

(P.T.O)

Q.4

(i) In a Bayesian estimation problem of the Poisson mean λ, a gamma prior (with density proportional to $e^{-\beta\lambda}\ \lambda^{\alpha-1}$) is formulated. There is a sample of size n from the Poisson and the sample mean is $\bar{x}$. Obtain the posterior distribution of λ and its mean. [7]

(ii) Let $X_1, X_2, \cdots, X_n$ be independent and identically distributed Bernoulli (θ), where $0<\theta<1$ and $n>1$. Let the priori density of θ be proportional to $\frac{1}{\sqrt{\theta(1-\theta)}}$, $0<\theta<1$. Define $S=\sum_{i=1}^{n} X_i$. Show that the posterior mean of θ exists and it is larger than the maximum likelihood estimator for some values of S. [7]

OR

(i) X has binomial distribution with parameters n and p. Suppose that n is given and the unknown parameter p has prior distribution, which is uniform on the interval [0, 1]. Consider the squared error loss function and the observation $X=n$. Obtain the Bayes estimate of p. Also find the median of the posterior distribution of p. [7]

(ii) Let $X \sim$ Poisson (θ), the prior distribution of θ be exponential with median $\log_e 2$ and the loss function be squared error. Obtain Bayes estimator of θ and posterior distribution. [7]

Q.5 Answer any seven: [14]

(i) Define sufficient statistic.

(ii) Define ancillary statistic.

(iii) Define complete sufficient statistic.

(iv) What do you understand by Bhattacharya bounds?

(v) The maximum possible form of reduction of sample values without loss of information will lead to

(a) sufficient statistic (b) minimal sufficient statistic

(c) maximal sufficient statistic (d) efficient statistic

(vi) MLE's are always consistent estimators but need not be unbiased.

(a) True (b) False

(vii) Which of the following statements are true about minimal sufficient statistic?

(a) It cannot be reduced further without loss of information

(b) It can be expressed as a function of every other sufficient statistic

(c) both (a) and (b)

(d) neither (a) nor (b)

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(viii) Let X_1, X_2,---, X_n be iid from $f(x,\theta)$, then $\prod_{i=1}^{n} f(x_i,\theta)$ is called ______

(a) Maximum likelihood function
(b) Likelihood function
(c) Marginal likelihood function
(d) Conditional likelihood function

(ix) If a sufficient estimator exists, it is a function of the maximum likelihood estimator.
(a) True (b) False

(x) Define proper and improper prior?

(xi) What do you understand by conjugate prior?

(xii) Let X_1, X_2 be i.i.d. random variables from an exponential distribution with mean $\frac{1}{\theta}$ where $\theta > 0$. Suppose that the prior distribution for θ is exponential with mean 1. Then the Bayes estimator for θ w.r.t. the squared loss error function is

(a) $\frac{X_1 + X_2}{2} + 1$ (b) $\frac{2}{X_1 + X_2}$ (c) $\frac{3}{X_1 + X_2 + 1}$ (d) none of the above

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