0402E1416

Candidate's Seat No :_________

MSc Sem.-1 Examination

403

AMS

February-2025

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Show that $x = 0$ is a regular singular point of $(2x + x^3)y'' - y' - 6xy = 0$ and find its solution about $x = 0$. **(07)**

(b) Describe the nature of the critical point of the system and sketch the trajectory: **(07)**

$$\frac{dx}{dt} = -x + 2y, \qquad \frac{dy}{dt} = x - y$$

OR

(a) For the initial value problem $\frac{dy}{dx} = y^2 + \cos^2 x, y(0) = 0$, determine the interval of existence of its solution given that R is the rectangle containing origin, **(07)**

$$R: \left\{(x, y):\ 0 \le x \le a,\ \ |y| \le b,\ \ a > \tfrac{1}{2},\ \ b > 0\right\}$$

(b) Find all the eigenvalues and eigenfunctions of the Strum-Liouville problem: **(07)**
$Y'' + \lambda Y = 0$ with $y(0) + y'(0) = 0$ and $y(1) + y'(1) = 0$

Q.2 **(a)** Find the general integral of the following linear PDE: $(y + zx)p - (x + yz)q = x^2 - y^2$ **(07)**

(b) I. Eliminate the arbitrary function from the following equation and hence, obtain the corresponding partial differential equation $xyz = \varphi(x + y + z)$ **(07)**

II. Form the partial differential equation by eliminating the constants from

$$z = ax + by + ct$$

OR

(a) Find the integral surface of the linear PDE: $xp - yq = z$, which contains circle: $x^2 + y^2 = 1, z = 1$. **(07)**

(b) Find the complete integral of the equation $(p^2 + q^2)y = qz$. **(07)**

Q.3 **(a)** Find the characteristics of the given PDEs: **(07)**

I. $y^2 r - x^2 t = 0$

II. $x^2 r + 2xys + y^2 t = 0$

(b) State (only) one-dimensional Homogeneous Wave equation (D'Alembert's solution). **(07)**
Solve: $u_{tt} = c^2 u_{xx}, -\infty < x < \infty, t > 0$
and $u(x, 0) = e^{-|x|}, u_t(x, 0) = \cos(4x), -\infty < x < \infty$

OR

(P.T.O)

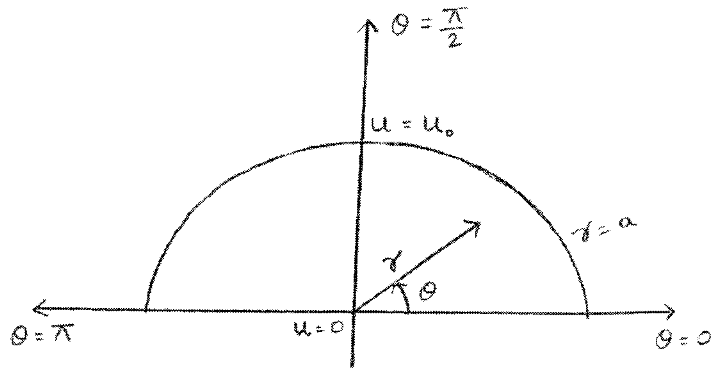
E1416-2

(a) Solve: $u_t = c^2 u_{xx}$, where c^2 is constant and $0 < x < l$ **(07)**

subject to condition: $u(0,t) = u(l,t) = 0, \forall x, t \geq 0 \; and \; u(x,0) = f(x), 0 < x < l$.

(b) Find the deflection of a vibrating string of unit length having fixed ends, with initial velocity zero and initial deflection $f(x) = a\sin^2 \pi x$. (Hint: Use D'Alembert solution) **(07)**

Q.4 (a) Find the steady state temperature distribution in a semi-circular plate of radius $'a'$, insulated on both the faces with a curved boundary kept at a constant temperature U_0 and its boundary diameter kept at zero temperature. **(07)**



Governing heat flow equation is $u_t = \nabla^2 u$

In steady state, temperature is independent of time, $u_t = 0$

To solve:

P.D.E.: $\nabla^2 u(r,\theta) = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$

B.C.s: $u(a,\theta) = U_0,\; u(r,\theta) = 0,\; u(r,\pi) = 0$

(b) **(i)** Prove that if the Dirichlet problem for a bounded region has a solution, then it is unique. **(07)**

(ii) Consider the parallel plate capacitor, where $V = 0$ at $z = 0$ and $V = 100v$ at $z = d$. Assuming the region between the plates charge-free. Calculate potential between the plates.

OR

(a) If u be a harmonic function in the interior of a rectangle $0 \leq x \leq a,\; 0 \leq y \leq b$ in the XY-plane satisfying Laplace equation **(07)**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with boundary conditions $u(0,y) = u(a,y) = u(x,b) = 0, u(x,0) = f(x)$

Obtain the solution to the above problem.

(b) Let $u(r, \theta)$ be the bounded solution of the following boundary value problem in polar coordinates: **(07)**

$$\nabla^2 u(r,\theta) = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0,\ 0 < r < 2$$
$$u(2,\theta) = \cos^2\theta\,, 0 \le \theta \le 2\pi$$

Then, find the value of $u\left(1,\frac{\pi}{2}\right) + u\left(1,\frac{\pi}{4}\right)$.

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Give any (two) harmonic function which satisfies the Laplace Equation.

(2) State (only) ordinary point of the differential equation with suitable example.

(3) Classify the nature of the PDE: $u_{xx} + u_{yy} + u_{zz} + u_{yz} + u_{zy} = 0$.

(4) Eliminate the arbitrary function from the following equation and hence, obtain the corresponding partial differential equation $x + y + z = f(x^2 + y^2 + z^2)$.

(5) Find the complete integral of the following linear PDE: $z = (x + y) + f(xy)$.

(6) Find the complete integral of the following linear PDE: $p^2 + q^2 = x + y$.

(7) State (only) The Maximum/Minimum Principle.

(8) Write the complementary function for the non-homogeneous PDE: $DD'(D - 2D' - 3)z = 0$.

(9) Find the characteristic curves for one dimensional Wave equation.

(10) The force of attraction F both inside and outside the attracting matter, can be expressed in terms of a Gravitational Potential $'u'$ by the equation $F = \nabla u$. In empty space, $'u'$ satisfies _______ equation.

(11) Find the complete integral of the following linear PDE: $z = px + qy - \log(pq)$.

(12) State (only) Neumann's problem for the rectangle.

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