0302E1364

Candidate's Seat No :_________

MSc Sem.-1 Examination

402

Data Science

February-2025

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 (a) Let V be the set of all ordered pairs of real numbers with vector addition defines as $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2 + 1, y_1 + y_2 + 1)$. Show that the first five axioms for vector addition are satisfied. Clearly mention the zero vector and additive inverse. **(07)**

(b) Define Subspace of a Vector Space. **(07)**

i. Check whether the set $W = \{(x, y) | x^2 + y^2 + z^2 = 1\}$ is a subspace of $\mathbb{R}^3$ with the standard operations or not?

ii. Show that the vectors $v_1 = (2,1,1), v_2 = (1,2,2), v_3 = (1,1,1)$ are linearly independent.

OR

(a) Let $\mathbb{R}^3$ have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis vectors $u_1 = (1,0,3),\ u_2 = (2,2,0),\ u_3 = (3,1,2)$ into an orthonormal basis. **(07)**

(b) Define Span of a Vector Space. **(07)**

Determine whether the given vectors $v_1 = (2,-1,3), v_2 = (4,1,2), v_3 = (8,-1,8)$ span $\mathbb{R}^3$.

Q.2 (a) Define Eigenvalues and Eigenvectors with its geometric interpretation. **(07)**

Find the Eigenvalues and Eigenvectors for the matrix $A = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$

(b) Determine Algebraic and Geometric Multiplicity of each eigenvalue of the matrix **(07)**

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

OR

(a) Define Linear Transformations. **(07)**

Show that the function $T: R^2 \to R^2$, where $T(x, y) = (x + 2y, 3x - y)$ is linear transformations.

(b) Show that the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ is not diagonalizable. **(07)**

(P.T.O)

Q.3 **(a)** Solve the following system of equations using partial pivoting by Gauss elimination method: **(07)**

$$8x_2 + 2x_3 = -7$$
$$3x_1 + 5x_2 + 2x_3 = 8$$
$$6x_1 + 2x_2 + 8x_3 = 26$$

(b) Obtain Cholesky factorization of the matrix **(07)**

$$\begin{bmatrix} 4 & 8 & 12 \\ 8 & 20 & 20 \\ 12 & 20 & 41 \end{bmatrix}$$

OR

(a) Solve the following system of linear equation by using Gauss-Siedel method: **(07)**

$$8x + y + z = 5$$
$$x + y + 8z = 5$$
$$x + 8y + z = 8$$

(b) Solve the system of equations by Gaussian elimination and back substitution. **(07)**

$$x_1 + x_2 + 2x_3 = 9$$
$$2x_1 + 4x_2 - 3x_3 = 1$$
$$3x_1 + 6x_2 - 5x_3 = 0$$

Q.4 **(a)** State Steepest Descent (Cauchy) method. **(07)**
Determine the minimum of the given function $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ using steepest descent method with initial guess $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

(b) Using Fibonacci Search method, Minimize the function $f(x) = x^2 + \frac{54}{x}$ in the interval $[0,5]$. Take $n = 3$. **(07)**

OR

(a) State Newton's method. **(07)**
Determine the minimum of the given function $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ using Newton's method with initial guess $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

(b) Write an algorithm of Golden Section Search Technique. **(07)**

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Define: Basis of a Vector Space.

(2) State any two techniques based on Region of Elimination.

(3) State Unimodal function. Explain its types with suitable graphs.

(4) What is the dimension of the vector space $\mathbb{R}^3$?

(5) State (only) Cayley-Hamilton theorem.

(6) Explain Well-conditoned and Ill-conditioned with suitable example.

(7) Find the trace of the matrix $A = \begin{bmatrix} 1 & -3 \\ 6 & -4 \end{bmatrix}$

(8) Let $T: R^3 \rightarrow R^3$ be a one-to-one linear transformation, then the dimension of $Ker(T)$ is

A. 0

B. 1

C. 2

D. 3

(9) Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, then the eigen values of A^2 is

A. 2,3

B. 4,9

C. 4,6

D. 3,6

(10) Define: Symmetric matrix with suitable example.

(11) The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is

A. 1

B. 2

C. 3

D. 9

(12) State (only) the necessary condition for applying the Gauss-Seidel iterative method.

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