5/31

0302E1358

Candidate's Seat No : _____

MSc Sem.-1 Examination

402

AMS

Time : 2-30 Hours] **February-2025** **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of a non-programmable scientific calculator is allowed.

Q.1 **(a)** A company needs to transport goods from two warehouses to two stores. Warehouse 1 has 100 units available, and Warehouse 2 has 150 units available. Store 1 requires 80 units, and Store 2 requires 170 units. The cost of transporting from Warehouse 1 to Store 1 is Rs. 2 per unit, to Store 2 is Rs. 4 per unit, from Warehouse 2 to Store 1 is Rs. 5 per unit, and to Store 2 is Rs. 1 per unit. How should the company allocate transportation to minimize the total cost? Formulate the problem. **(07)**

(b) Define Sensitivity Analysis and its importance in LPP. **(07)**

Maximize $Z = 3x_1 + 5x_2$

Subject to the constraints

$x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \leq 1$

$x_1, x_2 \geq 0$

Determine variations in $c_j, j = 1,2$ which are permitted without changing the optimal solution. What is the range of c_1 & c_2 so that optimal solution remains unchanged.

OR

(a) Define: Duality of the LPP and its Economic Interpretation. **(07)**

Write the Dual of the following LPP:

(i) Minimize $Z = 7x_1 + 3x_2 + 8x_3$

Subject to the constraints

$8x_1 + 2x_2 + x_3 \geq 3$

$3x_1 + 6x_2 + 4x_3 \geq 4$

$4x_1 + x_2 + 5x_3 \geq 1$

$x_1 + 5x_2 + 2x_3 \geq 7$

$x_1, x_2, x_3 \geq 0$

(ii) Maximize $Z = x_1 + 5x_2$

Subject to the constraints

$x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \geq 6$

$x_1, x_2 \geq 0$

(b) For the Linear Programming Problem, **(07)**

Max $Z = 3x_1 + 5x_2$ such that

$x_1 \leq 4,\ 3x_1 + 2x_2 \leq 18, x_1, x_2 \geq 0$, the optimal solution is given by

(P.T.O)

B.V.	x_1	x_2	s_1	s_2	solution
s_1	1	0	1	0	4
x_2	$\frac{3}{2}$	1	0	$\frac{1}{2}$	9
$z_j - c_j$	$\frac{9}{2}$	0	0	$\frac{5}{2}$	45

Find the range of b_1 and b_2 so that solution remains optimal feasible.

Q.2 **(a)** Four new computers, $C1, C2, C3$ and $C4$ are to be installed in computer lab. There are 5 places $A, B, C, D,$ and E available. Because of limited space $C2$ cannot be placed at C and $C3$ cannot be placed at A. The assignment cost of the computers is to be placed below. Find the optimal assignment schedule. **(07)**

	Persons				
	A	B	C	D	E
C1	4	6	10	5	6
C2	7	4	-	5	4
C3	-	6	9	6	2
C4	9	3	7	2	3

(b) Consider the following data for a project **(07)**

Activity	A	B	C	D	E	F	G	H
Predecessor	-	-	A	B	A	C, D	C, D, E	F
Duration	2	3	7	2	4	5	2	3

Find the critical path and the expected project completion time.

OR

(a) A small project is composed of activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending (j) node numbers. **(07)**

Activity i-j	**Estimated duration (in weeks)**		
	Optimistic	**Most Likely time**	**Pessimistic**
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

i. Draw the project network.

ii. Find the expected duration and variance for each activity. What is the expected project length?

iii. Calculate the variance and standard deviation of the project length. What is the probability that the project will be completed
   a. At least 4 weeks earlier than expected?
   b. No more than 4 weeks later than expected time?
   (Use $\varphi(1.33) = 0.4082$)

(b) Using the Stepping-Stone method, find the optimality: **(07)**

	D1	**D2**	**D3**	**Supply**
S1	2	7	4	5
S2	3	3	1	8
S3	5	4	7	7
S4	1	6	2	14
Demand	7	9	18	34

Q.3 **(a)** Derive an order–level lot–size model when the replenishment rate is infinite and shortages are allowed. **(07)**

(b) i. A 3 × 3 transition matrix P can capture this switching behaviour for three brands (Brand A, Brand B, and Brand C), where each entry P_{ij} represents the probability of a customer switching from brand i to brand j. **(07)**

Transition matrix

$$P = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$$

Apply brand switching models.

ii. Consider a system where accounts can be in three states:
   (a) **Current (paid on time)**
   (b) **Overdue (payment is late but recoverable)**
   (c) **Written Off (account is unlikely to be paid)** 3 × 3 transition matrix

$$P = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Analyze the probability of payments or predict the likelihood of overdue accounts using Account Receivable method.

OR

(a) Give the formula of the optimum order quantity (Define each symbol used) and determine **(07)**
(a) Optimum number of orders placed per year
(b) Optimum cycle time
(c) Optimum total cost
(d) Average inventory

(P.T.O)

(b) i. Define a Markov Chain and explain its key property known as the "Markov property." **(07)**

ii. Explain Transition Probability Matrix with Brand Switching example.

Q.4 **(a)** Customers arrive at a sales counter manned by a single person according to a Poisson process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. Find the waiting time of a customer. **(07)**

(b) Define **(07)**

a. Input Process
b. Capacity of the system
c. Service channels
d. Queue length

OR

(a) Prove that the arrival follows the Poisson distribution. **(07)**

(b) In two hours in the day clients at the government office stand 20 minutes, but the service time continues to remain 36 minutes, then calculate for this period **(07)**

1. the probability that the officer is free,
2. the average number of clients in the office.

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) State Ture or False: In a standard LP problem (ready to write in Simplex table), the number of basic variables equal to the number of equality constraints.

(2) State (only) the basic difference between CPM and PERT.

(3) State (only) Total Float and Independent Float.

(4) In the optimal solution, more than one empty cell has their opportunity cost as zero, it indicates

a. The solution is not optimal
b. The problem has alternate solution
c. Something wrong in the solution
d. The problem will have a cycle

(5) If an opportunity cost value is used for an unused cell to test optimality, it should be

a. equal to zero
b. most negative number
c. most positive number
d. any value

(6) If the earliest starting time for an activity is 8 weeks, the latest finish times is 37 weeks and the duration time of the activity is 11 weeks. Find total float.

(7) The inventory is _____ stock for an organization.
- a. Physical
- b. Buffer
- c. Safety
- d. Chemical

(8) The time of placing an order is known as _____
- a. cycle time.
- b. lead-time.
- c. horizon time.
- d. None of the above

(9) If the unit purchase cost decrease, the optimum order quantity _____
- a. increases.
- b. decreases.
- c. either increases or decreases.
- d. no change.

(10) A steady state exists in a queue if _____
- a. $\lambda > \mu$.
- b. $\lambda < \mu$.
- c. $\lambda \leq \mu$.
- d. $\lambda \geq \mu$.

(11) Queueing models measure the effect of _____
- a. random arrivals.
- b. random service.
- c. uncertainty on the behavior of the queueing system.
- d. length of the queue.

(12) When there are more than one servers, customer behavior in which he moves from one queue to another is known as _____
- a. Balking.
- b. Alternating.
- c. Reneging.
- d. Jockeying.

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