0102E1287 Candidate's Seat No :________

MSc Sem.-1 Examination

401

Statistics

Time : 2-30 Hours] **February-2025** **[Max. Marks : 70**

Q.1

(i) Show that the number of linearly independent solutions of the equation $AX = O$ is $(n-r)$, r being the rank of the $m \times n$ matrix A. [7]

(ii) Explain permutation with examples. [7]

OR

(i) Define determinant. Explain Laplace method of expansion of a determinant. [7]

(ii) Describe homogeneous and non-homogeneous linear equations. [7]

Q.2

(i) State and prove Frobenius Theorem. [7]

(ii) P, Q are non-singular matrices, show that if [7]

$A = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} P^{-1} & 0 \\ 0 & Q^{-1} \end{bmatrix}$

OR

(i) Show that, if A, B are two $m \times n$ and $n \times p$ matrices such that $AB = 0$ then $\rho(A) + \rho(B) \le n$. [7]

(ii) Let $L : n \times n$ be a non-singular matrix and $M : n \times r$ and $N : r \times n$ be two matrices such that $P = L + MN$ is a non-singular matrix. Show that [7]
$P^{-1} = L^{-1} - L^{-1}M(I_r + NL^{-1}M)^{-1}NL^{-1}$.

Q.3

(i) Show that if X_1 and X_2 are two eigen vectors corresponding to the eigen value λ of a matrix A then their linear combination $\alpha X_1 + \beta X_2$ where, α and β are non-zero scalars is also an eigen vector corresponding to the same eigen value. [7]

(ii) Show that the product of the characteristic roots of a square matrix of order n is equal to the determinant of the matrix. [7]

OR

(i) If A is non-singular matrix, prove that the eigen values of A^{-1} are the reciprocals of the eigen values of A. [7]

(ii) Show that the scalar λ is a characteristic root of the matrix A if and only if the matrix $(A - \lambda I)$ is singular. [7]

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Q.4

(i) Show that every square matrix can be uniquely expressed as the sum of a hermitian and a skew-hermitian matrix. [7]

(ii) Explain different types of quadratic forms. [7]

OR

(i) In usual notations, show that the necessary and sufficient condition for a square matrix A to be hermitian is that $A^* = A$. [7]

(ii) Explain reduction of quadratic form into sum of squares form with example. [7]

Q.5 Answer any seven: [14]

(i) If $\begin{vmatrix} x+1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} = 4$, then $x =$

(a) 0 (b) -1 (c) 2 (d) none of these

(ii) Define nullity of a matrix.

(iii) The rank of the unit matrix of order n is :

(a) $n-1$ (b) n (c) $n+1$ (d) n^2

(iv) If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then x =

(a) 1 (b) 2 (c) 1/2 (d) none of these

(v) Inconsistent equations have ------------- solution.

(vi) Rank (AA')=

(a) rank A (b) rank A' (c) 1 (d) none of these

(vii) At least one characteristic root of every singular matrix is equal to:

(a) 1 (b) -1 (c) 0 (d) ∞

(viii) If A is an orthogonal matrix, then

(a) $|A| = 0$ (b) $|A| = \pm 1$ (c) $|A| = \pm 2$ (d) none of these

(ix) Characteristic roots of an idempotent matrix are either --------- or --------.

(x) Define tranjugate of a matrix.

(xi) Define hermitian matrix.

(xii) Diagonal elements of a skew-hermitian matrix are either

(a) real or zero (b) real or non-zero (c) pure imaginary or zero (d) pure imaginary or non-zero

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