

FA-134

February-2025

M.Sc., Sem.-I

**MAT-401 : Mathematics
(Mathematical Statistics)
(New)****Time : 2:30 Hours]****[Max. Marks : 70**

- Instructions :** (1) All questions are compulsory.
(2) Use of non-programmable scientific calculator is allowed.

1. (a) State and Prove Chebyshev's Inequality. 7
(b) State (only) Bayes' Theorem. 7

Following aircraft accidents, a detailed investigation is conducted. The probability that an accident due to structural failure is correctly identified is 0.9 and the probability that an accident that is not due to structural failure being identified incorrectly as due to structural failure is 0.2. If 25% of all aircraft accidents are due to structural failure given that it has been diagnosed as due to structural failure.

OR

1. (a) Consider the following probability density function : 7

$$f_x(x) = \begin{cases} kx, & 0 \leq x < a \\ k(2a - x), & a \leq x \leq 2a \\ 0, & \text{otherwise} \end{cases}$$

- (i) Find the value of k for which f is a probability density function.
(ii) Find mean, variance and cumulative distribution function of X.
(b) Define Conditional, Marginal and Joint probabilities with suitable examples. 7
2. (a) The continuous random variable W has the PDF $f_w(w) = 12w^2(1 - w)$, $0 < w < 1$.

Calculate $P(W) < \frac{1}{2}$ and determine an expression for the CDF $f_w(w)$. Also

Calculate $E(W)$, $\text{Var}(W)$ 7

- (b) The probability distribution of a random variable X is given by 7

$$f_x(x) = \begin{cases} 1/2, & x = 0 \\ 1/4, & x = 1 \\ 1/8, & x = 2 \\ 1/8, & x = 3 \\ 0, & \text{otherwise} \end{cases}$$

- (i) Evaluate $E(X)$ and $V(X)$ using moment generating function.
(ii) If $Y = (X - 2)^2$, find the CDF for Y.

OR

2. (a) Explain Joint, Marginal and Conditional Expectations for Continuous random variable with suitable examples. 7

(b) Consider the probability distribution of the discrete random vector $[X_1, X_2]$, where X_1 represent the number of orders for Aspirin Tablets in August at the neighbourhood drugstore and X_2 represents the number of orders in September.

The joint distribution is shown in the table below : 7

$X_1 \backslash X_2$	51	52	53	54	55
51	0.06	0.05	0.05	0.01	0.01
52	0.07	0.05	0.01	0.01	0.01
53	0.05	0.10	0.10	0.05	0.05
54	0.05	0.02	0.01	0.01	0.03
55	0.05	0.06	0.05	0.01	0.03

(i) Find the marginal distribution.

(ii) Find the expected sales in September given that sales in August were either 51, 52, 53, 54, or 55.

3. (a) Suppose the amount of time it takes to assemble a plastic module range from 27 to 39 seconds and that assembly times are uniformly distributed. 7

(i) What is the probability that a given assembly will take between 30 and 35 seconds ?

(ii) What is the probability that a given assembly will take fewer than 30 seconds ?

(b) Derive Mean, Variance and Moment Generating Functions of Geometric distribution. 7

OR

3. (a) Explain relationship between Poisson Distribution and Exponential Distribution with necessary calculations. 7

(b) Suppose 18 major computer companies operate in the India and out of that 12 are located in Gujarat. If three computer companies are selected randomly from the entire list, what is the probability that one or more of the selected companies are located in the Gujarat ? 7

4. (a) State Normal approximation to Binomial distribution. 7

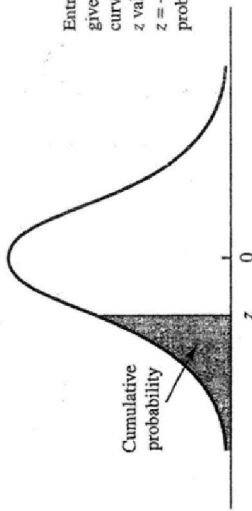
In sampling from production process that produces items of which 20% are defective, a random sample of 100 items is selected each hour of each production shift. The number of defectives in a sample is denoted by X. Find the probability of getting less than or equal to 15 defectives in a sample.

(b) Write Mean and Variance of Lognormal Distribution and mention its properties. Also, explain why lognormal distribution does not have moment generating function. 7

OR

4. (a) An assembly consists of three linkage components X_1 , X_2 and X_3 in series. Let $Y = X_1 + X_2 + X_3$. The properties of X_1 , X_2 and X_3 are given below, 7
- $$X_1 \sim N(12, 0.02)$$
- $$X_2 \sim N(24, 0.03)$$
- $$X_3 \sim N(18, 0.04)$$
- If X_1 , X_2 and X_3 are independent, determine $P(53.8 \leq Y \leq 54.2)$
- (b) Derive Mean and Variance of the Normal distribution and state its characteristics. 7
5. Attempt any **SEVEN** out of **TWELVE** : 14
- (1) State the Central Limit Theorem along with the necessary general conditions.
 - (2) Suppose X is a random variable such that $E(X) = 3$ and $V(X) = 5$. In addition, let $H(X) = 2X - 7$. Find $E[H(X)]$ and $V[H(X)]$.
 - (3) If X has a lognormal distribution with parameters μ_Y and σ_Y^2 and if a , b and are constants such that $b = e^d$, then $W = bX^a$ has a lognormal distribution with parameters _____ and _____.
 - (4) Define the moment-generating function if the probability distribution of a random variable X is exponentially distributed.
 - (5) Define : Skewness and Kurtosis.
 - (6) State Cauchy-Schwartz Inequality.
 - (7) Write Geometric distribution as a limiting case of Pascal distribution.
 - (8) The manager of a job shop does not know the probability distribution of the time required to complete an order. However, from past performance she has been able to estimate the mean and variance as 14 days and 2 (days)^2 , respectively. Find an interval such that the probability is at least 0.75 that an order is finished during that time.
 - (9) The random variable $Y = \ln X$ has a $N(10, 4)$ distribution. Find $E(X)$ and the mode of the distribution of X .
 - (10) Show that $M_x(t) = G_x(e^t)$ and $G_x(t) = M_x(\ln t)$.
 - (11) Define Probability generating function.
 - (12) Which are the distributions having the memoryless property ?
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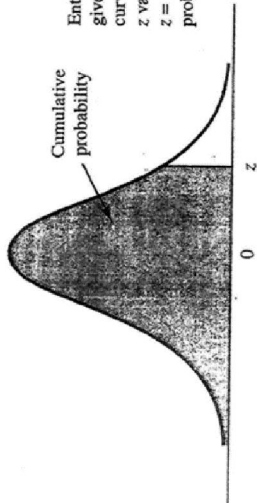
TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve to the left of the z value. For example, for $z = -.85$, the cumulative probability is .1977.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0020
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (Continued)



Entries in the table give the area under the curve to the left of the z value. For example, for $z = 1.25$, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990