

Instructions: All questions are compulsory. Use of non-programmable scientific calculators is allowed.

- Q.1 (a)** A medical test is used to detect a rare disease that affects 1 in 1,000 people. The test is 99% accurate. If a person has the disease, there is a 99% probability that the test will be positive (true positive rate). If a person does not have the disease, there is a 99% probability that the test will be negative (true negative rate). If a person tests positive, what is the probability that they actually have the disease? Using Bayes' Theorem. **(07)**

- (b)** Explain Moment Generating Function with properties. **(07)**

OR

- (a)** Consider a standard deck of 52 playing cards. What is the probability of drawing a Queen given that the card drawn is a face card (Jack, Queen, or King)? **(07)**

Total number of face cards = 12 (3 face cards per suit × 4 suits) and

Number of Queens = 4.

The probability of drawing a Queen given that the card is a face card?

- (b)** Explain Cumulative Distribution Function with properties. **(07)**

- Q.2 (a)** Find Probability Generating Functions of Poisson Distribution. **(07)**

- (b)** Let X be a continuous random variable with the probability density function (PDF) **(07)**

$$\begin{cases} 2 - x, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Derive the Moment Generating Function (MGF) of X .

OR

- (a)** Find Cumulant Generating Function of Normal Distribution. **(07)**

- (b)** Assume you have a dataset of returns on a particular investment. The average return is 10%, and the standard deviation is 4%. You want to determine the probability that a return falls within 2 standard deviations (10%) of the mean using Chebyshev's inequality. **(07)**

- Q.3 (a) Suppose X and Y are continuous random variables with a joint PDF defined as: (07)

$$f_{X,Y}(x, y) = \begin{cases} c \cdot (x + y) & \text{for } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant. Find c and verify that $f_{X,Y}(x, y)$ is a valid PDF.

Find Joint Probability Density Function and determine the value of c .

- (b) Find Mean and Variance of Poisson Distribution. (07)

OR

- (a) A group of students claims that their average test score is 75. However, a teacher suspects this is an overestimate. To test this, she takes a random sample of 10 students' scores, obtaining: (07)

72, 74, 78, 71, 73, 75, 76, 70, 74, and 73.

Table value is: 2.262

- (b) Find Mean and Variance of Uniform Distribution. (07)

- Q.4 (a) Consider two samples representing the weights (in grams) of a particular species of fruit from two different orchards: (07)

Orchard A: 150, 160, 165, 155, 170 and **Orchard B:** 180, 190, 200, 185, 195

Check that two populations have equal variances against the alternative hypothesis that they do not.

Table value is 6.338

- (b) Explain properties and find first two moments of Exponential Distribution. (07)

OR

- (a) Explain properties and find Mean and Variance of Lognormal Distribution (07)

- (b) In the pharmaceutical industry, the F-test is often used to compare the variances of two different batches of a drug or formulation to check if their variability is significantly different. (07)

A pharmaceutical company is testing the variance in the dissolution rates of two different drug batches:

- **Batch 1:** variance = 600, sample size = 8
- **Batch 2:** variance = 400, sample size = 10

Ftable value is 3.293

- Q.5** Attempt any **SEVEN** out of **TWELVE (True or False)** **(14)**
- (1) The Bernoulli distribution models a single trial with two possible outcomes: success and failure.
 - (2) Type II error occurs when the null hypothesis is incorrectly rejected when it is actually true.
 - (3) The t-distribution is used when the sample size is small and the population standard deviation is unknown.
 - (4) Chebyshev's inequality provides an upper bound on the probability that the value of a random variable deviates from its mean.
 - (5) When conducting a test of significance for the difference between two population means with known variances, the z-test is used.
 - (6) The F-distribution is used to compare the variances of two independent populations and is always positively skewed.
 - (7) If two random variables are independent, their joint probability density function equals the product of their marginal density functions.
 - (8) The expectation of a random variable is always equal to its median.
 - (9) Random sampling techniques ensure that every member of the population has an equal chance of being selected in the sample.
 - (10) The Hypergeometric distribution is used when sampling is done with replacement.
 - (11) The Normal distribution is characterized by its bell-shaped curve and is defined by its mean and variance.
 - (12) The Chi-square distribution is used primarily in hypothesis testing and is always symmetric.
 - (13) The Probability Density Function (PDF) can take negative values.
 - (14) The Cumulative Distribution Function (CDF) of a random variable is non-decreasing.
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