1601E1155

Candidate's Seat No :________

MSc AIML Sem.-1 Examination
Mathematical Foundations
January-2025

Time : 3-00 Hours] **[Max. Marks : 100**

Instructions:

- **Write both the Sections in the separate answer book.**
- **Both Sections having equal weightage.**
- **Draw Diagrams wherever necessary.**
- **Make Assumptions wherever necessary.**

SECTION – I

Q-1 Attempt **ALL** of the following questions: (21)

(A) In survey of blood donation camp of 100 people, it was found that 50 people have blood group A, 40 people have blood group B and 30 people have blood group O, 10 people have both blood group A and B, 5 people have both blood group B and O, 8 people have both blood group A and O and 3 people have all three blood groups. Answer the following:

1) How many people have exactly two blood groups?
2) How many people do not have any of the blood groups A, B or O?
3) How many people belong to blood group A only?
4) How many people belong to blood group A and B, but not blood group O?

(B) Find the equation of a line through the intersection of $x - y - 1 = 0$ and $2x - 3y + 1 = 0$. Also, find the followings:

1) Having slope -2
2) Parallel to the line $x + y + 4 = 0$
3) Passes through (1,2)

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(C) Examine the continuity of the following function:

$$f(x) = \begin{cases} -2x^2, & if\ x \leq 0 \\ 5x+2, & if\ 0 < x \leq 1 \\ 3x^2+4x, & if\ 1 < x \leq 2 \end{cases}$$

Q-2 Attempt any of the **THREE** questions: (15)

(A) Find the co-ordinates of the point which divides the line segment joining the points $(6,3)$ and $(-4,5)$ in the ratio $3:2$ (i) internally and (ii) externally.

(B) Let $A = \{1,2,4,5\}, B = \{3,5,7\}$ and $C = \{2,3,6\}$. Verify the followings:

i. $(A \cup B) - C = (A - C) \cup (B - C)$

ii. $(A \, \Delta \, B) - C = (A \ \cup C)\Delta(B \cup C)$

iii. $(A - B) \cup C = (A \cup C) - \ (B - C)$

(C) Evaluate the following Limits:

i. $\lim\limits_{x \to 0} \dfrac{(15)^x - 5^x - 3^x + 1}{x^2}$ ii) $\lim\limits_{x \to 0} \left(\dfrac{1-2x}{1+2x}\right)^{\frac{1}{3x}}$

(D) Find distance between each other and projection of $u = 2i - 3j + 5k$ on $v = 7i - 8j + k$.

(E) Determine linear transformation $T: \mathbb{R}^2 \dashrightarrow \mathbb{R}^2$ satisfying $T(1,0) = (1,1)$ and $T(0,1) = (-1,2)$.

Q-3 Attempt any of the **SEVEN** questions: (14)

a) Write down roaster form of a set $A = \{x \in \mathbb{Z} |\ x \text{ is a positive divisor of } 15\}$

b) If $U = \{a,b,c,d,e,f\}$, $A = \{a,b,c\}$ and $B = \{c,\ d,e,f\}$. Find bit string for the sets $A \, \Delta \, B$ and $A - B$ with respect U.

c) For which value of x the area of the triangle formed by the vertices $(x,4), (8,2)$ and $(6,7)$ is 13 units?

d) Find the equation of the line passing through the points $(2,3)$ having slope $-\frac{1}{2}$.

e) Define one-one function with example.

f) Evaluate: $\lim_{x \to -1} \frac{x^5 + 1}{x + 1}$

g) Derive derivative of $\sin x + x^4 + e^x$.

h) Define norm of vector. Calculate norm of $-j + 5k$.

i) Define angle between co-ordinate axes in $\mathbb{R}^2$.

SECTION – II

Q-4 Attempt **ALL** of the following questions: (21)

(A) Use the Gram-Schmidt process to orthonormalize the set of given linearly independent vectors $\{(1,0,1,1), (-1,0,-1,1), (0,-1,1,1)\}$.

(B) Show that the $\mathbb{R}^2$ is a vector space over $\mathbb{R}$ under component-wise vector addition and scalar multiplication $\alpha . v = (\alpha x, \alpha^3 y)$, for every $\alpha \in \mathbb{R}$ and $v = (x, y) \in \mathbb{R}^2$.

(C) Find three positive numbers whose sum is 36 and their product is maximum.

Q-5 Attempt any of the **THREE** questions: (15)

(A) Let $u = (2, -1, 3), v = (4, 2, 1), w = (1, 5, -2)$. Check whether the following vectors are members of $span[\{u, v, w\}]$ or not:

1) $10i + 4j + 7j$ 2) $7i + 3j + 9k$

(B) Check whether the following subsets of vector space are subspace or not:

$S_0 = \{(x, y) \in \mathbb{R}^2 | y = x\}$ 2) $S_1 = \{p \in \mathcal{P}_n | \, p(0) = 1\} \subset \mathcal{P}_n$

(C) Find Change-of-basis matrices from B to B' and B' to B for given two bases $B = \{(1,0,0), (0,1,0), (0,0,1)\}$ and $B' = \{(1,0,1), (2,1,2), (1,2,2)\}$.

(D) Find Directional Derivatives of following functions at mentioned point $\bar{a}$ along the direction $\bar{u}$:

$$f(x) = \begin{cases} \frac{x}{y+1}, & if \; y \geq 0 \\ \frac{y}{x+1}, & if \; y < 0 \end{cases} \quad \text{at } \bar{a} = (2,1) \text{ along } \bar{u} = (1,2)$$

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(E) If $u_1 = \frac{x^2+y^2}{z}, u_2 = \frac{yz}{x}, u_3 = \frac{zx}{y}$, then prove that $\frac{\partial(u_1,u_2,u_3)}{\partial(x,y,z)} = 6$.

Q-6 Attempt any of the **SEVEN** questions: (14)

a) Give an example of orthonormal set in $\mathbb{R}^3$.

b) Define orthogonal complement and give example in $\mathbb{R}^2$.

c) Define a subspace of vector space.

d) Give an example of non-linear transformation in $\mathbb{R}^2$.

e) Find the determinant of Jacobian matrix $\frac{\partial(x,y)}{\partial(r,\theta)}$ for $x = r^3 + \theta$, $y = r^2\theta$.

f) Define partial derivative of second order.

g) Normalize the vector $i + 2j - 3k$.

h) Find project of vector $u = (0,1)$ on $v = (1,1)$.

i) Define critical point for function of two variables.

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