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1601E1156

Candidate's Seat No : _____

MSc AIML (Rep) Sem.-2 Examination

Numerical Optimization

January-2025

Time : 3-00 Hours]

[Max. Marks : 100

Instructions:

- Write both the Sections in the separate answer book.
- Both Sections having equal weightage.
- Draw Diagrams wherever necessary.
- Make Assumptions wherever necessary.

SECTION – I

Q-1 Attempt **all** of the following questions: (18)

- (A) Consider unconstrained one dimensional optimization problem (10)
for $f: \mathbb{R} \rightarrow \mathbb{R}$ with $f \in C^2$. State and prove necessary and sufficient conditions. Give examples in which necessary condition fails to be sufficient and vice-versa.

OR

- (A) Explain Golden section search with golden section rule in (10)
details. Using golden section rule find minimum value of
 $f(x) = x^2 - 2.6x + 2$, $x \in [-2,3]$ up to 6 iterations.
- (B) Discuss the Fibonacci search method for optimization. Apply (08)
Fibonacci search method to determine minimum value of
 $f(x) = (x - 3)^2 + 2$ over the interval $[-2, 10]$ with $n = 6$.

Q-2 Attempt any of the **two** questions: (16)

- (A) Define Hessian Matrix. Find gradient vector and Hessian (08)
matrix for Rosenbrock function.
- (B) Using analytical method, find the extreme values of the (08)
function $f(x, y) = x^3 + y^3 - 3x - 12y + 5$.
- (C) Discuss any two optimization problem with mathematical (08)
formulation.

- Q-3 Attempt **all** of the following questions: **(16)**
- (A) Define truncated Taylor's series expansion with example. (03)
- (B) Define affine set with example. (03)
- (C) Define a convex function with example. (03)
- (D) Define epigraph of function defined on \mathbb{R} . (03)
- (E) Write down first order necessary condition of unconstrained minimization in terms of gradient vector. (02)
- (F) Find the decent direction of $f(x, y) = x^2 + y^2$ at a point $(1, 2)$. (02)

SECTION - II

- Q-4 Attempt **all** of the following questions: **(18)**
- (A) Write the algorithm for Steepest decent method and further use it to minimize $f(x, y) = 3x^2 + 2y^2 - 4xy + 5$ starting from $(1, 2)$. Perform three iterations. (10)

OR

- (A) Use Conjugate gradient method to minimize for the (10)
 $f(x, y) = x^2 + 3y^2 - 2xy + 2x - 4y + 5$, $x_0 = (1, 1)$
- (B) Explain the Fletcher- Roove's method in detail with example. (08)

- Q-5 Attempt any of the **two** questions: **(16)**
- (A) Write the algorithm for interior penalty method and further use it to Minimize $f(x, y) = x^2 + y^2$ subject to constraint $x + y - 1 \geq 0$. (08)
- (B) Use exterior penalty method to minimize $f(x, y) = (x - 2)^2 + (y - 1)^2$ subject to $g(x, y) = y - x^2 \geq 0$. (08)
- (C) Use Sequential quadratic programming algorithm to minimize $f(x, y) = x^2 + 2y^2$ subject to conditions are $g(x, y) = x + y - 1 \geq 0$, $h(x, y) = -x + y - 1 \geq 0$ starting with $(0, 0)$. Perform two iterations. (08)

- Q-6 Attempt **all** of the following questions: **(16)**
- (A) Find the gradient of the function $f(x, y) = x^3y + xy^2z - z^3$ at a point $(1, 1, 1)$. (03)
- (B) Give an example of 2×2 positive definite matrix. Also, give an example of 2×2 negative definite matrix. (03)

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(C) Without using Graph paper, graphically solve the following: (03)

$Min Z = 3x + 5y$, subject to constraints $x + 2y \leq 10$, $x + y \leq 7$,
where $x, y \geq 0$.

(D) Calculate inverse of a matrix (03)

$$\begin{bmatrix} 5 & -2 \\ -1 & 1 \end{bmatrix}$$

(E) What is the use of Gradient? (02)

(F) Write flow chart of Genetic algorithm. (02)

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