Seat No. : _______________

NO-124

November-2025

B.Sc., Sem.-III

MDC-C-MAT-234 T : Mathematics

(Elementary Vector Space and Permutations)

Time : 1:00 Hour] **[Max. Marks : 25**

Instructions : (1) **All** questions are compulsory.

(2) Write the question number in your answer book as shown in the question paper.

(3) The figure to the right indicates marks of the question.

1. (a) Prove that the intersection of two subspaces of a vector space V is also a subspace of V. **5**

1. (b) Show that $A = \{p(x) \in P_2(\mathbb{R}) : p'(1) = 0\}$ is a subspace of $P_2(\mathbb{R})$. **5**

OR

1. (a) Prove that in any vector space V, **5**

   (i) $\alpha\theta = \theta$, for every scalar α,

   (ii) $0u = \theta$, for every vector $u \in V$.

1. (b) Show that the set $A = \{(x_1, x_2, x_3) \mid x_1 = 2x_2]$ is a subspace of R^3. **5**

2. (a) Prove that any two disjoint cycles in S_n are commutative. **5**

2. (b) Obtain the order of the following permutations : **5**

   (i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 10 & 7 & 8 & 14 & 9 & 12 & 5 & 11 & 6 & 3 & 4 & 1 & 13 & 2 \end{pmatrix} \in S_{14}$

   (ii) $(2, 1, 3, 4, 5, 6)(1, 7, 2) \in S_7$.

OR

2. (a) Define a symmetric group S_n of degree n and prove that the order of S_n is n ! **5**

2. (b) (i) Prove or disprove : $(1, 2, 3, 4)\ (3, 1, 2, 7)\ (5, 8)\ (8, 2, 1, 6) \in S_8$ is even. **5**

(ii) For $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 2 & 3 & 1 & 6 \end{pmatrix} \in S_6$, find $g^{-1} f$.

3. Attempt any **Five** out of **Six** in short : **5**

(i) Define subspace of a vector space.

(ii) Is the set $\{(x_1, x_2) : x_1 = 1\}$ a subspace of R^2 ? Justify !

(iii) Is the set of all positive integers $\mathbb{N}$, with usual addition and scalar multiplication a vector space over $\mathbb{R}$?

(iv) What is the order of A_{2025} (an alternative group of degree 2025) ?

(v) Express the identity permutation in S_4 as a product of transpositions.

(vi) Define the quotient group.