Seat No. : ______________

NN-126

November-2025

B.Sc., Sem.-III

DSC-CC-232 (Major) : Mathematics

(Group Theory)

Time : 2:00 Hours] **[Max. Marks : 50**

Instruction : All questions are compulsory and carry **10** marks.

1. (a) Define a relation "congruent modulo n" in the set of integers. Show that it is an equivalence. **5**

1. (b) Let $Z_5 = \{[0], [1], [2], [3], [4]\}$ then show that $(Z_5, +_5)$ is a group. **5**

OR

1. (a) Let " * " be the binary operation defined on the set $G = \mathbb{R} - \{-1\}$ as $a * b = a + b + ab$. Show that (G, *) is an abelian group. **5**

1. (b) Give an example of a finite noncommutative group of least order with justification. **5**

2. (a) Define the order of an element 'a' of the group G.
If G be a finite group of order n then prove that for $a \in G$, there exists a positive integer $r \leq n$ such that $a^r = e$, also prove that $O(a) \leq O(G)$. **5**

2. (b) Show that if $a^2 = e$ for each element a of a group G, then G is commutative group. **5**

OR

2. (a) State and prove the Lagrange's theorem for a finite group. **5**

2. (b) Prove : a subset $H \neq \phi$ of a group G is subgroup of G if for $a, b \in H \Rightarrow ab^{-1} \in H$. **5**

3. (a) If H is normal subgroups of group G then show that $H_aH_b = H_{ab}$ for a, b $\in$ G. **5**

3. (b) Define : Symmetric group of degree n. **5**

Write all elements of S_3. Is S_3 commutative ? Is S_3 cyclic ?

OR

3. (a) Define : odd and even permutation. **5**

Prove : for n $\geq$ 2, the set of all even permutation in S_n is a subgroup of order $^{n!}/_2$

3. (b) State all proper normal subgroups of a group $(Z_{15}, +_{15})$, also find Index of each. **5**

4. (a) State and prove the first Fundamental theorem of Homomorphism. **5**

4. (b) Is $(Z_{24}, +_{24})$ cyclic ? If it is cyclic then find all generators and possible subgroups of it. **5**

OR

4. (a) Show that any subgroup of cyclic group is cyclic. **5**

4. (b) Define automorphism on G and obtain all automorphism on G = V_4-Klein's group. **5**

5. Attempt any **TEN** in short : **10**

(a) Determine x so that $3x \equiv 1 \pmod 6$.

(b) Define cyclic subgroup.

(c) If $a^2 = e$ for each element a of a group G then G is commutative : True or False ?

(d) Solve the equation $x +_7 [6] = [\,1\,]$ in Z_7.

(e) Define : Quotient Group with Illustration

(f) Define : Transposition and even permutation.

(g) Give an example of a group which is isomorphic to all of its subgroups.

(h) If 0(G) = 7 then state all the elements of G and find the order of each element of G.

(i) If G = $\langle a \rangle$ and 0(G) = 12 then find $0(a^8)$.

(j) If G = (2 6 5 1) (2 3 5 4) (1 2 3 4 5) then find 0(G).

(k) Give an example of non-abelian group having all its subgroups are normal subgroups.

(l) Define : Kernel of homomorphism. What is the kernel of 1-1 hoinomorphism ?