Seat No. : _____

NM-103

November-2025

B.Sc., Sem.-III

DSC-C-MAT-231T : Mathematics

(Linear Algebra-I)
(As per NEP 2020)
(Major)

Time : 2:00 Hours] **[Max. Marks : 50**

Instructions : (1) **All** questions are compulsory and carry equal marks.
(2) Symbols and notations are usual.

1. (A) If (p, q) is a solution of $Dx^2 + m = y^2$, then prove that $D(2pq)^2 + m^2 = (Dp^2 + q^2)^2$. **5**

1. (B) Divide : $(5 + \sqrt{90} + \sqrt{180} + \sqrt{648})$ by $(\sqrt{5} + \sqrt{36})$. **5**

OR

1. (A) Find approximate value of $\sqrt{10}$. **5**

1. (B) Evaluate $\sqrt{112} + \sqrt{28}$ by using identity $\sqrt{a} + \sqrt{b} = \sqrt{\frac{1}{c}(\sqrt{ac} + \sqrt{bc})^2}$. **5**

2. (A) Prove that $\mathbb{R}^2$ is a vector space under the operations of vector addition $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$ and scalar multiplication $(x_1, x_2) = (\alpha x_1, \alpha x_2)$, for $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$ and α is scalar. **5**

2. (B) Find the dimension of the subspace spanned by vectors (1, 1, 0), (1, 2, 1), (2, 3, 1), (0, –1, –1). **5**

OR

2. (A) If dim V = n, then any subset of V containing n linearly independent vectors becomes a basis of V. **5**

2. (B) Let B = {(1, 2, 1), (1, 0, 3), (1, 1, 1)}. Prove that B is a subspace of $\mathbb{R}^3$ and obtain co-ordinate vector of $x = (1, –1, 3)$ relative to ordered basis B. **5**

3. (A) Prove that $T : U \to V$ is a non-singular linear map, then $T^{-1} : V \to U$ is a linear, one-one and onto map. **5**

3. (B) Verify Rank-Nullity theorem for the linear map $T : \mathbb{R}^3 \to \mathbb{R}^3$, defined as $T(x, y, z) = (x + y + z, y + z, z)$, $\forall (x, y, z) \in \mathbb{R}^3$. **5**

OR

3. (A) Let T : U $\rightarrow$ V be a linear transformation, prove that T is one-one if and only if $N(T) = \{\theta_U\}$. **5**

3. (B) Define: Linear Transformation. Check whether the map T : $\mathbb{R}^3 \rightarrow \mathbb{R}^3$, $T(x, y, z) = (x + z, y + z, x + y)$, $\forall (x, y, z) \in \mathbb{R}^3$ is linear or not. **5**

4. (A) Let $M_{m\times n}(\mathbb{R})$ be the set of all m $\times$ n type of real matrices. Prove that the dimension of a vector space $M_{m\times n}(\mathbb{R})$ is mn. **5**

4. (B) Find associated matrix $[T ; B_1, B_2]$ of the linear map T : $\mathbb{R}^3 \rightarrow \mathbb{R}^3$,

$T(x, y, z) = (x + y, y + z, z + x)$ with respect to the ordered bases

$B_1 = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$, $B_2 = \{e_1, e_2, e_3\}$ of $\mathbb{R}^3$. **5**

OR

4. (A) Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$. Find linear transformation associated to matrix A relative to $B_1 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $B_2 = \{(1, 1), (1, -1)\}$ of $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively. **5**

4. (B) Find the rank and nullity of the matrix $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$. **5**

5. Do as directed : (Attempt any **TEN** in short) **10**

(i) State Bramhgupta's Identity.

(ii) Using Vedic method evaluate $\sqrt{8} + \sqrt{2}$.

(iii) What is the dimension of vector space $P_2(x)$?

(iv) Show that $(3,5) \in [(1, 1), (1, 0)]$.

(v) Define Subspace of a vector space.

(vi) True / False : The set $\{(1, -2), (0, 0)\}$ is linear independent in $\mathbb{R}^2$.

(vii) Define Range space of a linear map T : U $\rightarrow$ V.

(viii) If $T(x, y, z) = (x + z, y - z, z)$, find $T(1, 2, -1)$.

(ix) If $r(T) = 2$ of a linear map T : $\mathbb{R}^4 \rightarrow \mathbb{R}^5$, then $n(T) =$ _____.

(x) Write rank of identity matrix of $M_p(\mathbb{R})$.

(xi) Dimension of space $L(\mathbb{R}^3, \mathbb{R}^2)$ is _____.

(xii) If linear transformation T : $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined by $T(x, y) = (y, x)$ with respect to the ordered bases $B_1 = B_2 = \{e_1, e_2\}$ of $\mathbb{R}^2$, then obtain matrix associated with T.
