Seat No. : _______________

NH-123

November-2025

B.Sc., Sem.-V

DSC-M-MAT-354 T (Minor) : Mathematics

(Fourier Series and Complex Variables)

Time : 2:00 Hours] **[Max. Marks : 50**

Instructions : (1) **All** questions are compulsory.

(2) Write the question number in your answer book as shown in the question paper.

(3) The figure to the right indicates marks of the question.

1. (a) Find the Fourier series of $f(x) = x^2, x \in (-\pi, \pi)$ and $f(x + 2\pi) = f(x)$. **5**

1. (b) Let $f : (0, 2\pi) \to R$ be defined as

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$$ **5**

$f(x + 2\pi) = f(x)$. Obtain the Fourier series of f.

OR

1. (a) Let $f : (0, 2\pi) \to R$ be defined as **5**

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ 2\pi - x, & \pi < x < 2\pi \end{cases}$$

$f(x + 2\pi) = f(x)$. Obtain the Fourier series of f.

1. (b) Find the Fourier series of $f(x) = 2x - x^2, x \in (0, 3)$. **5**

2. (a) Let $z_1, z_2 \in C$ be given. Prove that $|z_1 + z_2| \le |z_1| + |z_2|$. **5**

2. (b) Let $\theta \in R$. Prove that $(\cos\theta + i\sin\theta)^n = \cos n\theta + \sin n\theta, n \in N$. **5**

OR

2. (a) Find the cube root of $-8i$. **5**

2. (b) Solve : $e^z = -1$. **5**

3. (a) Prove that when limit of function exists at a point z_0, it is unique. **5**

3. (b) Let $f : C \to C$ be defined as

$$f(z) = \begin{cases} \dfrac{\text{Re}\, z}{|z|}, & z \ne 0 \\ 0, & z = 0 \end{cases}$$ Show that $\lim_{z \to 0} f(z)$ does not exist. **5**

OR

3. (a) Show that $\lim\limits_{z \to 0} \frac{\bar{z}}{z}$ does not exist. **5**

3. (b) Let $f : C \to C$, be defined as $f(z) = z^2$, $z \in C$. Show that f is differentiable on C. Find its derivative. **5**

4. (a) Discuss the image of the line $ax + by + c = 0$ under the transformation $w = \frac{1}{z}$. **5**

4. (b) Find bilinear transformation that maps points –1, 0, 1 on to the points –i, 1, i. **5**

OR

4. (a) Find the image of the infinite strip $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. **5**

4. (b) Determine the linear fractional transformation that maps 0, 1, ∞ on to –1, –i, 1. **5**

5. Answer in brief : (Any **Ten**) **10**

(i) Define : Fourier series of function

(ii) Define : Fourier sine series

(iii) Let $f : [-a, a] \to R$ be an even function. Find $\int_{-a}^{a} f(x)dx$.

(iv) Evaluate : $\int_{-\pi}^{\pi} x^4 \sin x \, dx$.

(v) Define : Polar form of a complex number.

(vi) Define : Conjugate of a complex number.

(vii) Sketch the region : $|z - 1| < 1$.

(viii) Write down C-R equations in Cartesian form.

(ix) Define : Harmonic function.

(x) Define : Bilinear transformation.

(xi) Discuss the Conformality of $f(z) = e^z$.

(xii) Find the critical points of $f(z) = z^2 - 1$.