

Time : 2-30 Hours]

1. (A) Let $f(x, y, z) = x^2 e^{x-y+3z}$. Compute the differential df . Use the differential to estimate the difference $f(1.1, 1.2, -0.1) - f(1, 1, 0)$. 7
- (B) The two planes $x + z = 4$ and $3x - y = 6$ intersect in a line L . Use Lagrange's method to find the point on L that is closest to the origin. 7

OR

- (A) Find and classify the critical points of the function $f(x, y) = x^2 y^2 (2 - x - y)$. 7
- (B) Find the Taylor polynomial of order 4 based at $\mathbf{a} = (0, 0)$ for the following function:
 $f(x, y) = x \sin(x + y)$. 7
2. (A) Let $(u, v) = f(x, y) = (x - 2y, 2x - y)$. Then compute the inverse transformation $(x, y) = f^{-1}(u, v)$. Also find the image in the uv - plane of the triangle bounded by the lines $y = x$, $y = -x$, and $y = 1 - 2x$. 7
- (B) Investigate the possibility of solving the equations

$$\begin{cases} xy + 2yz - 3xz = 0 \\ xyz + x - y = 1 \end{cases}$$

for two of the variables as functions of the third near the point $(x, y, z) = (1, 1, 1)$. 7

OR

- (A) Let S be the circle formed by intersecting the plane $x + z = 1$ with the sphere $x^2 + y^2 + z^2 = 1$. Find a parametrization of S . Find parametric equations for the tangent line to S at the point $(\frac{1}{2}, -\frac{1}{\sqrt{2}}, \frac{1}{2})$. 7
- (B) Let $(u, v) = \mathbf{f}(x, y) = (x^2 + 2xy + y^2, 2x + 2y)$. Compute the Jacobian $\det D\mathbf{f}$. Draw a sketch of the images of some of the lines $x = \text{constant}$ and $y = \text{constant}$ in the uv -plane. Find a formula for a local inverse of \mathbf{f} . 7

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3. (A) Find the centroid of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{3} + \frac{y}{5} + \frac{z}{7} = 1$. 7
- (B) An object with mass density $\rho(x, y, z) = yz$ occupies the cube $\{(x, y, z) : 0 \leq x, y, z \leq 2\}$. Find its mass and center of mass. 7

OR

- (A) Find the volume of the region above the triangle in the xy -plane with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$, and below the surface $z = 6xy(1 - x - y)$. 7
- (B) Find the volume of the portion of the sphere $x^2 + y^2 + z^2 = 4$ lying above the plane $z = 1/2$. 7
4. (A) Compute $\int_C \bar{F} \cdot dX$, where $\bar{F}(x, y) = (x - y, x + y)$ and C is the circle $x^2 + y^2 = 1$, oriented clockwise. 7
- (B) Let S be the annulus $1 \leq x^2 + y^2 \leq 4$. Compute $\int_{\partial S} (xy^2 dy - x^2 y dx)$ by using the Green's theorem. 7

OR

- (A) Find the area of the part of the surface $z = x^2 + y^2$ inside the cylinder $x^2 + y^2 = a^2$. 7
- (B) Compute $\iint_C (x^2 + y^2) dA$, where S is the portion of the sphere $x^2 + y^2 + z^2 = 4$ with $z \geq 1$. 7
5. **Attempt any seven of the following.** 14

- (1) Compute ∇f for the function $f(x, y) = \frac{x+2y+4}{7x+3y}$.
- (2) Find the directional derivative of the function $f(x, y) = x^2 + 5xy^2$ at the point $(-2, 1)$ in the direction $(1, 0)$.
- (3) Find critical points of the function $f(x, y) = x^2 + y^2 + 2x + 2y$.
- (4) Let $w = f(2x - y^2, x \sin 3y, x^4)$. Find $\partial_x w$ in terms of the partial derivatives $\partial_1 f$, $\partial_2 f$ and $\partial_3 f$.
- (5) Let $(u, v) = f(x, y, z) = (2x + y^3, xe^{5y-7z})$. Then compute $Df(x, y, z)$.
- (6) Let $F = (f, g, h)$, where $f(x, y, z) = x + 2y - z$, $g(x, y, z) = x - 3y + 4z$, and $h(x, y, z) = 2x - y + 3z$, then find rank of DF .

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- (7) Determine whether the set $\{(x, y) : x^2 - 3y^2 - 3 = 0\}$ is a smooth curve.
 - (8) Express iterated integral $\int_1^2 \int_0^{\log x} f(x, y) dy dx$ as iterated integral in opposite order.
 - (9) Compute Laplacian of the function $f(x, y) = x^5 - 10x^3y^2 + 5xy^4$.
 - (10) Compute the divergence of the vector field $F(x, y, z) = (xy^2, xy, xy)$.
 - (11) Find arc length of the curve $f(t) = (\frac{1}{3}t^3 - t, t^2)$, $t \in [0, 2]$.
 - (12) State Green's theorem.
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Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

- Q.1 (a)** Explain in detail the comparison of NPV and IRR with suitable example. (07)
- (b)** Assume the following cash flows from 2 projects: (07)

Year	Project A	Project B
0	-1000	-1000
1	500	100
2	400	300
3	300	400
4	100	600

Assume that the cash flows are occurring at the end of each year. Find out the internal rate of return for their projects. The cost of capital is 10%.

OR

- Q.1 (a)** Consider two investments with the following sets of cash flows: (07)

End of period	P (in Rs.)	Q (in Rs.)
0	-10	-20
1	4.2	0
2	4.2	0
3	4.2	26

What is the crossover rate for these two investments profiles?

- (b)** Explain in brief the Macaulay duration and write an expression which establishes the relation between Macaulay and Modified duration. (07)
- Q.2 (a)** Explain the Markowitz theory indicating its assumptions, efficient portfolio, efficient frontier, optimal portfolio and the limitations of the theory. (07)
- (b)** Consider two stocks, A and B, such that $\sigma_A = 0.30$, $\sigma_B = 0.80$, $\bar{R}_A = 0.10$, $\bar{R}_B = 0.06$ and $r_f = 0.02$. (07)
- What is the minimum variance portfolio when $\rho_{AB} = 0$ and what is its volatility?
 - What is the minimum variance portfolio when $\rho_{AB} = 0.6$ and what is its volatility?
 - What is the minimum variance portfolio when $\rho_{AB} = -0.6$ and what is its volatility?

OR

- Q.2 (a)** Write detailed theory on: The Capital Asset Pricing Model (CAPM) (07)
- (b)** Consider a world with only two risky assets, A and B, and a risk-free asset. Stock A has 200 shares outstanding, a price per share of \$3.00, an expected return of 16% and a (07)

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volatility of 30%. Stock B has 300 shares outstanding, a price per share of \$4.00, an expected return of 10% and a volatility of 15%. The correlation coefficient $\rho_{AB} = 0.4$. Assume CAPM holds.

- a) What is expected return of the market portfolio?
- b) What is volatility of the market portfolio?
- c) What is the beta of each stock?
- d) What is the risk-free rate?

- Q.3** (a) Suppose there is a Forward contract on an asset providing known income whose current price is \$900. Income \$40 occurs at 4-month; 4-month and 9-month interest rates are 3% and 4% respectively. Maturity of contract is 9 months. If Forward price is \$870, then check is there any arbitrage opportunities in this situation? If yes, calculate the risk-free profit of Arbitrageur. (07)
- (b) You are an option dealer. Given binomial model for stock prices, you sell call option with $S_0 = 50$, $S_u = 60$, $S_d = 40$, $X = 55$, $r = 0.05$, $T = 0.5$ and number of shares are 1000. (07)
- a) Calculate the fair market price for the call option.
 - b) Assume you sell 1000 shares of the option at the fair market price + 0.10. How many shares of stock you buy to hedge the sell?
 - c) What is your profit, independent of the outcome of the stock price?

OR

- Q.3** (a) Consider a 10-month forward contract on a stock when the stock price is \$50. We assume that the risk-free rate of interest is 8% per annum for all maturities. We also assume that dividends of \$0.75 per share are expected after 3 months, 6 months, and 9 months. Then calculate the present value of the dividend (I) and forward price (F_0). (07)
- (b) Find the value of European call option for the following data by using two step binomial option pricing model. $S_0 = 50$, $X = 55$, $u = 1.2$, $d = 0.8$, $r = 0.065$, $T = 0.5$ (07)
- Q.4** (a) Derive the Black-Scholes-Merton formula for a European call option. (07)
- (b) If the volatility of a stock is 18% per annum, estimate the standard deviation of the percentage price change in (07)
- a) 1 day
 - b) 1 week
 - c) 1 month

OR

- Q.4** (a) Derive the expression of Ito's lemma. (07)
- (b) What is the price of European call option on a non-dividend paying stock when the stock price is \$52, the strike price is \$50, the risk-free interest rate is 12% per annum, the volatility is 30% per annum and time to maturity is three months? Use following table. (07)

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Z	0.38	0.39	0.53	0.54
$P(0 < x < Z)$	0.1480	0.1517	0.2019	0.2054

Q.5 Attempt any **SEVEN** out of **TWELVE**:

(14)

- (1) What is Net Present Value?
- (2) Explain the relation between price and yield.
- (3) Explain clean and dirty price.
- (4) Define variance of the return.
- (5) What is the meaning of market beta?
- (6) Define efficient frontier.
- (7) What is the basic difference between Forward and Futures contracts?
- (8) Define: call option with payoff graph.
- (9) What is role of Δ in delta hedging method?
- (10) What is implied Volatility?
- (11) What is Gamma of call/put option?
- (12) What is Put - Call Parity?
