Seat No. : _____

NG-112

November-2025

B.Sc., Sem.-V

DSC-C-MAT-352 T (Major) : Mathematics

(Complex Analysis and IKS)

Time : 2:00 Hours] **[Max. Marks : 50**

Instructions : (1) **All** questions are compulsory.
(2) Figures to the right indicate full marks of the question/sub-question.
(3) Notations used in this question paper carry their usual meaning.

1. (A) Explain the Vedic method of algebraic division using the sutra परावत्यर् योजयेत् (Parāvartya Yojayet). **5**

(B) Find the HCF (using Lopana-Sthāpanābhyām) for
$P(x) = x^2 - 4$, $Q(x) = x^2 + 5x + 6$. **5**

OR

1. (A) Find the quotient and remainder when $P(x) = x^3 + 3x^2 + 5x + 7$ is divided by $D(x) = x + 2$ using the Vedic method Parāvartya Yojayet. **5**

(B) Factorise the expression $x^2 + 10x + 16$ using the Vedic Sutra Anurupyena (अनुरुप्येण). **5**

2. (A) Prove that, if z_1 and z_2 are complex numbers then $|z_1 + z_2| \leq |z_1| + |z_2|$. **5**

(B) Find all values of $(-8i)^{\frac{1}{3}}$. **5**

OR

2. (A) Suppose that $z_n = x_n + iy_n (n = 1, 2, 3, \ldots)$ and $z = x + iy$. Then prove that $\lim_{n \to \infty} z_n = z \Leftrightarrow \lim_{n \to \infty} x_n = x$ and $\lim_{n \to \infty} y_n = y$. **5**

(B) If $\sin(\alpha + i\beta) = x + iy$, then prove that **5**

(i) $\dfrac{x^2}{\cos h^2 \beta} + \dfrac{y^2}{\cos h^2 \beta} = 1$

(ii) $\dfrac{x^2}{\sin^2 \alpha} - \dfrac{y^2}{\cos^2 \alpha} = 1.$

3. (A) Prove that if a function $f(z) = u(x, y) + i\, v(x, y)$ is differentiable at a point $z = (x, y)$, then u_x, u_y, v_x, v_y exist at that point and they satisfy C-R equations $u_x = v_y$ and $u_y = -v_x$ at that point. **5**

(B) Find the harmonic conjugate of the function $u(x, y) = y^3 - 3x^2y$ and the corresponding analytic function. **5**

OR

3. (A) Prove that $f(z) = z^2$ is analytic everywhere and find its derivative $f'(z)$. **5**

(B) If $u - v = e^x(\cos y - \sin y)$ is a function, then find out corresponding analytic function, where "v" is a harmonic conjugate of "u". **5**

4. (A) Find the image of the infinite strip $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. Show them graphically. **5**

(B) Find a bilinear transformation that maps the points ∞, $-I$, 0 of z-plane in to 0, 1, ∞ of w-plane respectively. **5**

OR

4. (A) Prove that if $f(z)$ is analytic at z_0 and $f'(z_0) \neq 0$, then $w = f(z)$ is conformal at z_0. **5**

(B) If $w = f(z)$ is a bilinear transformation which associate z_1, z_2, z_3 and z_4 to w_1, w_2, w_3 and w_4 respectively, then prove that $\frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_4)(z_3 - z_2)} = \frac{(w_1 - w_2)(w_3 - w_4)}{(w_1 - w_4)(w_3 - w_2)}$. **5**

5. Answer the following questions in short : (Any **ten**) **10**

(1) Find the product 14×12 using the Vedic method Urdhva-Tiryagbhyam.

(2) Which Vedic method is used to find factorization of a polynomial ?

(3) Which Vedic method is used to find LCM of two polynomials ?

(4) Find real and imaginary parts of $\frac{1}{1+i}$.

(5) Prove that $\tan ix = i \tan h\, x$.

(6) State De Moivre's theorem.

(7) Define harmonic function.

(8) If a complex function f is differentiate at a point z_0, is it necessary that it is analytic at that point ?

(9) State C-R equations in Polar form.

(10) Find the angle of rotation produced by the transformation $w = \frac{1}{z}$ at point $z = 1$.

(11) Define isogonal transformation.

(12) True/False : The mapping $w = \frac{1}{z}$ transforms the circles and lines into circles and lines.