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1311E1103

M.Sc. Sem.-3 Examination

501

Statistics

November-2025

Candidate's Seat No :_____

Time : 2-30 Hours] **[Max. Marks : 70**

Note: Attempt all questions.

Q.1

(i) Define Test Function, Randomised Test Function and Non-Randomised Test Function. [7]

(ii) Show that a test obtained by N-P lemma to test $H:\theta=\theta_0$ against $K:\theta=\theta_1$ is always unbiased. [7]

OR

(i) State and prove N-P fundamental lemma for randomized test. [7]

(ii) Let $f(x,\theta)=\frac{1}{\theta}e^{-x/\theta};\ x>0,\theta>0$. Check whether distribution possess MLR property in $T(X)$ or not. [7]

Q.2

(i) Define Likelihood Ratio Test . Under what circumstances would you recommend this test? State its properties. [7]

(ii) What do you understand by nuisance parameter? Let $X_1,X_2,\cdots,X_n$ be a random sample from $N(\mu,\sigma^2)$, where μ is unknown. To test $H:\sigma\le\sigma_0$ against $K:\sigma>\sigma_0$, derive UMP test of size α and power function of the test. [7]

OR

(i) Let $X\sim f(x,\theta),\theta\in\Omega$. To test $H:\theta=\theta_0$ against $K:\theta\neq\theta_0$, θ_0 is known. If $\lambda(x)$ is the test criterion given by LRT then show that the distribution of $-2\ln\lambda(x)\sim\chi^2_{(1)}$ for large n, where n = sample size. [7]

(ii) Let $X_1,X_2,\cdots,X_n$ be a random sample from $U(0,\theta)$, $\theta>0$. Obtain $(1-\alpha)100\%$ UMA CI for θ. [7]

Q.3

(i) Describe Wald's Sequential Probability Ratio Test. [7]

(ii) Give the S.P.R.T. for testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1(>\theta_0)$, in sampling from a normal density

$$f(x,\theta)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[\frac{-1}{2}\left(\frac{x-\theta}{\sigma}\right)^2\right],\quad -\infty<x<\infty$$

where σ is known. Also obtain its A.S.N. function. [7]

OR

P.T.O

E1103-2

(i) Let X have the distribution

$f(x,\theta)=\theta^x(1-\theta)^{1-x}\quad ;x=0,1;0<\theta<1$

For testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1$, construct S.P.R.T. and obtain its OC function. [7]

(ii) Develop S.P.R.T. for testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1(>\theta_0)$, where θ is the parameter of a Poisson distribution. Find approximate expressions for its OC function. [7]

Q.4

(i) Explain Sigel-Tukey test. [7]

(ii) Explain Kruskal-Wallies one-way ANOVA test. [7]

OR

(i) Explain Kolmogorov-Smirnov one-sample test. [7]

(ii) Explain Friedman's two- way analysis of variance test. [7]

Q.5 Answer any seven: [14]

(i) Which of the following is the defining characteristic of the critical region in the Neyman-Pearson lemma?

(A) It is defined by the likelihood ratio being less than or equal to a constant (k).

(B) It is defined by the likelihood ratio being greater than or equal to a constant (k).

(C) It is the region where the likelihood function under H_1 is greater than the likelihood function under H_0.

(D) It is the region where the probability of a Type I error is minimized.

(ii) Neyman-Pearson lemma is used for finding most powerful test for

(A) simple vs simple hypothesis

(B) simple vs composite hypothesis

(C) composite vs simple hypothesis

(D) composite vs composite hypothesis

(iii) Define UMPU test.

(iv) A test is said to have the Neyman structure with respect to a sufficient statistic T if:

(A) The power of the test is maximized for all values of T.

(B) The expected value of the test's critical function, conditional on T, is constant under the null hypotheses.

(C) The test statistic is a function of the sufficient statistic T only.

(D) The Type II error rate is minimized, given a fixed Type I error also.

(v) The primary application of tests having the Neyman structure is in the construction of:

(A) Uniformly Most Powerful (UMP) tests for simple hypotheses.

(B) Consistent estimators.

(C) Uniformly Most Powerful Unbiased (UMPU) tests for composite hypotheses.

(D) Confidence intervals using the general method.

(vi) The concept of Neyman structure is particularly useful when dealing with:
(A) Non-parametric tests (B) Exponential family distributions
(C) Descriptive statistics (D) Bayesian estimation

(vii) What is the primary purpose of the Sequential Probability Ratio Test?
(A) To determine the exact fixed sample size needed for a hypothesis test.
(B) To make a decision on a hypothesis by sequentially evaluating data as it is collected.
(C) To find the probability of a Type II error for a given Type I error rate.
(D) To plot the likelihood ratio against the sample number.

(viii) What is a significant advantage of the SPRT compared to a fixed sample size test?
(A) It guarantees a smaller sample size in all cases.
(B) It controls for Type I and Type II errors with fewer samples on average, particularly when the true parameter value is far from the null hypothesis.
(C) It is not affected by the distribution of the data.
(D) It eliminates the need for a likelihood ratio.

(ix) Sequential Probability Ratio Test of the general linear hypothesis terminates with probability
(A) 0 (B) 0.5 (C) 1 (D) none of the above

(x) The Kruskal-Wallies test is the non parametric alternative to the _____.
(A) Factorial design (B) One-way ANOVA (C) Two-way ANOVA (D) none of the above

(xi) Give one advantage of non parametric test.

(xii) Which of the following is a key characteristic of non-parametric test?
(A) They require data to be normally distributed.
(B) They are generally more powerful then parametric tests.
(C) They are less strict about data assumptions, making them suitable for ordinal data.
(D) They are designed to be used with continuous data only.

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