1311E1102 Candidate's Seat No :________

M.Sc. Sem.-3 Examination

501

Mathematics

Time : 2-30 Hours] **November-2025** **[Max. Marks : 70**

1. (A) If L is a linear space and M is a linear subspace of L then define the quotient space L/M. When do we get $L/M = \{0\}$? Give details. **7**

   (B) Give two linear subspaces M_1 and M_2 of linear space $C[a,b]$ such that $dim(M_1) = 1$ and $dim(M_2) = 2$. Explain. **7**

OR

1. (A) Let L be a linear space and B be the smallest spanning subset of L, show that B is a basis for L. **7**

   (B) Give an example (with all the details) of an infinite-dimensional linear space. **7**

2. (A) Let $T : N \to N'$ be a linear map. If T is continuous then prove that there exists a $K \geq 0$ such that $||T(x)|| \leq K||x||$, for all $x \in N$. **7**

   (B) Prove that $\mathbb{R}^n$ is a Banach space with respect to any norm $||\cdot||$ on it. **7**

OR

2. (A) Show that $C[0,1]$ is normed linear space with the norm $||\cdot||_1 = \int_0^1 |f|dx$. Is it a Banach space (don't give proof)? **7**

   (B) Show that the norms $||\cdot||_1$, $||\cdot||_2$ and $||\cdot||_\infty$ on $\mathbb{R}^n$ are equivalent. **7**

3. (A) If N is a normed linear space then show that N can be imbedded in its second conjugate space N^{**}. **7**

   (B) State and prove the closed graph theorem. **7**

OR

3. (A) Define a projection on a Banach space B. If P is a projection on a Banach space B, with range M and the null space N, prove that $B = M \oplus N$. 7

   (B) Let B be a Banach space and $T : B \to B$ be one-one, onto, linear, continuous transformation, prove that T^{-1} is continuous. **7**

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E-1102.2

4. (A) Give an example of a collection of functions defined on some set X which are pointwise bounded but not uniformly bounded on X. Give details. **7**

(B) Let $T \in \beta(N)$, define its conjugate $T^* \in \beta(N^*)$.
Show that $(T_1T_2)^* = T_2^*T_1.^*$ **7**

OR

4. (A) Define an orthonormal set in a Hilbert space. Prove that any orthonormal set is linearly independent. Is the converse true? **7**

(B) Let H be a Hilbert space and $S \subset H$. Define the orthogonal complement $S^{\perp}$ of S . Show that $S^{\perp}$ is always a closed and linear subspace of H. **7**

5. **Attempt any SEVEN of the following:** **14**

(1) What is the dimension of the linear space $C[a, b]$ over $\mathbb{R}$?

(A) finite
(B) n
(C) countable infinite
(D) uncountable infinite

(2) Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x+4y, 3x+7y)$. Then ________

(A) T is linear
(B) T is one one
(C) T is onto
(D) T is not invertible

(3) Let $L = P_n$, the linear space of all polynomials (over $\mathbb{R}$) of degree less than n. Which of the following are norms on L?

(A) $||p|| = sup\{|p'(t)|/t \in [0, 1]\}$
(B) $||p|| = \int_0^1 |p(t)|dt$
(C) $||p|| = sup\{|p(t)|/t \in [0, 1]\}$
(D) none of above

(4) What is the dimension of the linear space $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$?

(A) infinite (B) 2 (C) 1 (D) 4

(5) Consider the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ defined by $T(x_1, x_2, x_3, x_4) = (x_3, x_4, x_1, x_2)$. Then ________

(A) T is linear
(B) T is continuous
(C) T is non-linear
(D) T is discontinuous

E 1102-3

(6) Which of the following subsets of $\mathbb{R}$ is/are complete?

(A) $\{1, 2, 3, 4, 5\}$

(B) $\{\frac{1}{n}/n \in \mathbb{N}\}$

(C) $[0, 2\pi)$

(D) $\mathbb{Q}(\sqrt{2})$

(7) Let N be a normed linear space with the norm $||\cdot||$ .Then the function norm $||\cdot||$ on N is ____ function

(A) linear

(B) continuous

(C) differentiable

(D) uniformly continuous

(8) If M is linear subsapce of a real linear space L. Then M is ________

(A) finite dimensional

(B) closed

(C) convex

(D) none of the above

(9) The set $\mathbb{R}$ with the usual metric is homeomorphic to ________

(A) $\mathbb{Q}$

(B) $[0, 1]$

(C) $[0, 2\pi)$

(D) $(0, 2\pi)$

(10) Which of the following is a Hilbert space?

(A) $\mathbb{R}^n$ with $||\cdot||_1$

(B) $C[0, 1]$ with $||\cdot||_1$

(C) $\mathbb{R}^n$ with $||\cdot||_2$

(D) none of the above

(11) Which of the following is false?

(A) $(l_2)^* = l_2$

(B) $C[0, 1]^* = C[a, b]$

(C) $(\mathbb{R}^n)^* = \mathbb{R}^n$

(D) none of the above

(12) Which of the following normed linear space is not complete?

(A) $P[a, b]$ with norm $||\cdot||_2$

(B) $\mathbb{C}^2$ with norm $||\cdot||_2$

(C) l_1^n

(D) none of the above