

Seat No. : \_\_\_\_\_

# AD-123

April-2025

B.Sc., Sem.-VI

MAT-308 : Mathematics

(Analysis-II)

Time : 2:30 Hours]

[Max. Marks : 70

- Instructions :** (1) All questions are compulsory.  
(2) Write the question number in your answer sheet as shown in the question paper.  
(3) Figures to the right indicate marks of the question.

1. (a) State and prove that every continuous function on  $[a, b]$  is Riemann integrable. 7  
(b) Prove that  $\frac{2\pi^2}{9} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{2x}{\sin x} dx \leq \frac{4\pi^2}{9}$ . 7

**OR**

1. (a) State and prove First Mean Value Theorem of Integral Calculus. 7  
(b) Prove that  $f(x) = 3x^2$  is Riemann integral on  $[0, 1]$ . 7  
2. (a) State and prove Cauchy's condensation test. 7  
(b) Prove that the series  $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$  converges for  $p > 1$  and it diverges for  $p \leq 1$ . 7

**OR**

2. (a) Prove that a complex sequence converges if and only if its real part and imaginary part converge. 7  
(b) Discuss the convergence of the following series : 7  
(i)  $\sum_{n=1}^{\infty} \frac{n}{5^n}$   
(ii)  $\sum_{n=1}^{\infty} \frac{1}{n^{2025} + 2025}$   
(iii)  $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

3. (a) State and prove Mertens' theorem for the Cauchy product of two series. 7  
 (b) Discuss the convergence of the following series for different values of  $z \in \mathbb{C}$ . 7  
 (i)  $\sum_{n=1}^{\infty} n(n+1)z^n$  (ii)  $\sum_{n=1}^{\infty} \frac{z^n}{n!}$

**OR**

3. (a) Let  $a_n \in \mathbb{C}$ . If  $\sum a_n$  is absolutely convergent, then prove that it is convergent but the converse is not true. 7  
 (b) Let  $a_n > 0$  and  $\sum a_n$  be divergent series. Then prove that  $\sum \frac{a_n}{1+na_n}$  diverges. 7

4. (a) If  $m$  is a positive integer then prove that  $(1+x)^m = 1 + \frac{m}{1!}x + \frac{m(m-1)}{2!}x^2 + \dots + \frac{m(m-1)(m-2)\dots(m-n+1)}{n!}x^n$ . Where  $-\infty < x < \infty$ . 7

- (b) Derive Taylor's series with the integral form of remainder for  $f(x) = \cos x$  about  $a = 0$  in  $(-\infty, \infty)$ . 7

**OR**

4. (a) State and prove the Taylor's series. 7  
 (b) Obtain the power series solution of the differential equation  $(1-x)y' + 1 = 0$  with condition  $y(0) = 1$ . 7

5. Attempt any **seven** in short : 14

- (i) If  $f(x) = \begin{cases} 1, & 1 \leq x \leq 2 \\ 2, & 2 < x \leq 3 \end{cases}$  then prove that  $f$  is Riemann integral on  $[1, 3]$ .  
 (ii) If  $P = \{1, 4, 5, 7, 8, 10\}$  be partition of  $[1, 10]$  then find  $\|P\|$ .  
 (iii) Give an example of function which is bounded on  $[a, b]$  but not Riemann integrable.  
 (iv) Prove that  $\int_1^{\infty} \frac{1}{x^2} dx$  is convergent.  
 (v) Discuss the convergence of the series  $\sum (-1)^n$ .  
 (vi) State Cauchy Root Test.  
 (vii) Define bounded sequence.  
 (viii) Find the radius of convergence of  $\sum a_n z^n$  if  $\limsup |a_n|^{\frac{1}{n}} = 0$ .  
 (ix) Define absolutely convergent series.  
 (x) Prove that  $\sum \frac{(-1)^n}{n}$  is conditionally convergent.  
 (xi) Write the power series of  $e^x$  for any real  $x$ .  
 (xii) Can we expand the series of function  $f(x) = \sqrt{x}$  using Taylor's Maclaurin series ?