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1104E138

Candidate's Seat No :_____

M.Sc. Sem.-4 Examination

507

Mathematics

April-2025

Time : 2-30 Hours] **[Max. Marks : 70**

1. (A) Let $\overline{\alpha}(t) = (\cos^3 t,\ \sin^3 t,\ \cos 2t)$. Find the arc-length of $\overline{\alpha}$ for $\frac{\pi}{6} \leqslant t \leqslant \frac{\pi}{3}$. Does $\overline{\alpha}$ define a regular curve for $0 < t < 2\pi$? Explain. **7**

   (B) Define the torsion κ_2 of a regular curve at a point. Prove that $|\kappa_2| = |\overline{\beta}'|$. **7**

OR

1. (A) Let $\overline{\alpha}(s)$ be a curve with natural parametrization and curvature $\kappa_1(s) \neq 0$. If $\overline{\alpha}$ lies in a plane P, show that its osculating plane is (P. hence) same. Show that this conclusion fails for nonplanar curves. **7**

   (B) Show that the tangents to a helix $x = a\cos\omega t,\ y = a\sin\omega t,\ z = bt$ make a constant angle with the $xy-$ plane. **7**

2. (A) Investigate the nature of points on quadric surfaces. **7**

   (B) Find the equation of the surface obtained by rotating a curve $\overline{\alpha}(t) = (f(t),\ 0,\ g(t))$ about the z-axis. **7**

OR

2. (A) Make up the equation of the surface formed by straight lines parallel to a vector $\overline{v_0}$, and intersecting a curve $\overline{r}(u)$. **7**

   (B) Prove that if all normals to a surface are concurrent, then the surface is a part of a sphere. **7**

3. (A) Find the normal curvature of $x^2 + y^2 + z^2 = 1$ at the point $(1, 0, 0)$ in the direction $(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ **7**

   (B) Find the second fundamental form of helicoid. **7**

OR

3. (A) Define asymptotic line on a surface. Find a family of asymptotic lines on helicoid. **7**

   (B) Show that the absolute value of the Gaussian curvature can be obtained as the limit of ratio of two areas. **7**

(P.T.O)

4. (A) Show that an asymptotic geodesic line is straight. **7**

   (B) State(without proof) the second version (piecewise smooth curves version) of Gauss Bonnet theorem. Define the characteristic of a surface. **7**

OR

4. (A) Define geodesic line and line of curvature on a surface. Show that if a geodesic line is a line of curvature then it lies in plane. **7**

   (B) Prove that the sum of all three interior angles of a geodesic triangle on a surface with negative Gaussian curvature is less than π. **7**

5. **Attempt any SEVEN of the following:** **14**

(1) If $\overline{\alpha}$ is a unit speed curve, then $[\overline{\alpha}', \overline{\alpha}'', \overline{\alpha}'''] =$ _____

(A) $\kappa_1\kappa_2$ (C) $\kappa_1^2\kappa_2^2$

(B) $\kappa_1\kappa_2^2$ (D) $\kappa_1^2\kappa_2$

(2) What is parametric representation of the curve $x^3 + y^3 - 3xy = 0$?

(A) $x = \frac{3t}{1-t^3}$, $y = \frac{3t^2}{1-t^3}$ (C) $x = \frac{-3t}{1-t^3}$, $y = \frac{3t^2}{1-t^3}$

(B) $x = \frac{3t}{1+t^3}$, $y = \frac{3t^2}{1+t^3}$ (D) $x = \frac{3t}{1+t^3}$, $y = \frac{-3t^2}{1+t^3}$

(3) Which of the following curve is a cycloid?

(A) $x = (t - \sin t)$, $y = (1 + \cos t)$ (C) $x = (t - \sin t)$, $y = (1 - \cos t)$

(B) $x = (t + \sin t)$, $y = (1 + \cos t)$ (D) $x = (t + \sin t)$, $y = (1 - \cos t)$

(4) Which of the following equations represents a hyperboloid of one sheet?

(A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{c^2}{z^2} - 1 = 0$ (C) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{c^2}{z^2} + 1 = 0$

(B) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{c^2}{z^2} + 1 = 0$ (D) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{c^2}{z^2} - 1 = 0$

(5) What is the Dupin indicatrix of a surface at a parabolic point?

(A) An ellipse (C) A hyperbolic paraboloid

(B) Two conjugate hyperbolas (D) Two parallel lines

(6) Let $\overline{r}(u, v) = (\cosh u \cos v, \cos h \sin v, u)$. The given surface is _____

(A) helicoid (C) catenoid

(B) ellipsoid (D) hyperbolic paraboloid

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(7) The normal curvature of a surface along _____ direction vanishes.

(A) line of curvature
(B) asymptotic
(C) principal
(D) geodesic line

(8) Identify the surface $z = x^2 + y^2$.

(A) ellipsoid
(B) paraboloid
(C) hyperboloid
(D) cylinder

(9) The Gaussian curvature of the circular cylinder of radius a is _____

(A) $\frac{1}{a^2}$ (B) $\frac{1}{a}$ (C) 0 (D) $\frac{-1}{a^2}$

(10) The shortest path on the surface $x^2 + y^2 + z^2 = 2$, between the points $(1, 0, 1)$ and $(-1, 0, -1)$ has length

(A) 0 (B) $\sqrt{2}\pi$ (C) $2\sqrt{2}\pi$ (D) 2π

(11) Which of the following formulas give the Gaussian curvature K?

(A) $(LN - M^2)/(EG - F^2)$
(B) $(LM - N^2)/(EG - F^2)$
(C) $(LN - M^2)/(F^2 - EG)$
(D) $(LM - N^2)/(EG - F^2)$

(12) A loxodrome cuts each line of _____ at some fixed angle.

(A) latitude
(B) equator
(C) parallel
(D) longitude