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Candidate's Seat No :_____

B.Sc. Semester-5 Examination

SE 305

Mathematics (A)

March-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Que-1. (a) Let $(L, \leq)$ be lattice. Prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$ [9]

(b) Let $(L, *, \oplus)$ be a lattice. For any $a \in L$. Prove that [9]

(i) $a * a = a$

(ii) $a \oplus a = a$

OR

Que-1 (a) Let $(L, \leq)$ be a lattice and $a, b \in L$. Then prove that $a \leq b \implies a * b = a$. [9]

(b) Let $(L, \leq)$ be a lattice and $a, b \in L$. Prove that $a \oplus (a * b) = a$. [9]

Que-2 (a) State and prove De Morgan's laws in a Boolean algebra. [9]

(b) Show for Boolean algebra and prove that $a = 0 \Leftrightarrow ab' + a'b = b$. [9]

OR

Que-2 (a) In Boolean algebra $(B, *, \oplus, 0, 1)$ prove that $(a \oplus b) = a * b$ [9]

(b) Obtain the product of sums canonicals form in three variables x_1, x_2, x_3 of the Boolean expression $x_1 * x_2$. [9]

Que-3 (a) Prove that the sum of all min terms in n-variables is 1. [9]

(b) Draw Hasse diagram of $< S_{30}, D >$ and $< S_{60}, D >$. [9]

OR

Que-3 (a) Show that in a Boolean algebra B, for any $a, b \in B$ $a = b \Leftrightarrow (a * b') \oplus (a' * b) = 0$ [9]

(b) Define a Boolean algebra and give one example of a Boolean algebra. [9]

Que-4 Attempt any eight. in short [16]

(a) Define *reflective property.*

(b) Define *anti-symmetric property.*

(c) Define *least upper bound.*

(d) Define *homomorphism.*

(e) Define *Lattice.*

(f) Define *bounded lattice.*

(g) Define *sub lattice* with one example.

(h) Define *complemented lattice.*

(i) Define *atom* of a Boolean algebra.

(j) Define *partially order relation.*

(k) Define *Boolean expression.*

(l) Define *min terms.*

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B.Sc. SEMESTER-V(Elective Course)
MATHEMATICS
MAT-305 (

Instructions: (1) Each question are compulsory.
(2) Figures to right indicate full marks to the question.

Q-1 (A) State and prove Division algorithm theorem. [9]

(B) Using the Euclidean algorithm to obtain the integer x and y such that $\gcd(12378, 3054) = 12378x + 3054y$. [9]

OR

Q-1 (A) Prove that there are infinite number of primes of the form $4n+3$. [9]

(B) Define Linear Diophantine equation. Find the solution of linear Diophantine Equation $54x + 21y = 906$ in positive integers. [9]

Q-2 (A) Let $n > 0$ be fixed and a, b, c are integers then prove that if $a \equiv b(\text{mod } n)$, $c \equiv d(\text{mod } n) \Rightarrow ac \equiv bd(\text{mod } n)$ and $a^k \equiv b^k(\text{mod } n)$ for any positive integer k. [9]

(B) Does there exists a solution of the congruence $15x \equiv 9(\text{mod } 12)$? If so, find out all mutually congruent solution of it. [9]

OR

Q-2 (A) Define congruence relations and prove that it is an equivalence relation. [9]

(B) Using Chinese remainder theorem , find integer x such that $2x \equiv 1(\text{mod } 3)$ $3x \equiv 1(\text{mod } 5)$; $5x \equiv 1(\text{mod } 7)$. [9]

Q-3 (A) State and prove Wilson's theorem. [9]

(B) If p and q are distinct primes such that $a^p \equiv a(\text{mod } q)$ and $a^q \equiv a(\text{mod } p)$ then prove that $a^{pq} \equiv a(\text{mod } pq)$ [9]

OR

Q-3 (A) State and prove the Euler's theorem. [9]

(B) In usual notation show that $(1835^{1910} + 1986^{2061}) \equiv 0(\text{mod } 7)$. [9]

Q-4 Attempt any **EIGHT (In short):** [16]

(1) If p is a prime number and p/ab then prove that p/a or p/b.

(2) Is Diophantine equation $2x + 6y = 9$ has solution? Justify your answer.

(3) A number 360 can be written as product of prime in canonical form.

(4) Solve: $18x \equiv 30(\text{mod } 42)$.

(5) Define complete residue system modulo.

(6) If $g.c.d(a,b) = d$ then prove that $g.c.d\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

(7) Prove that $n^4 + 4$ is composite number for $n \in N$ with $n > 1$.

(8) Prove that the number $N = 1571724$ is divisible by 9 and 11.

(9) Prove that n is prime if and only if $\phi(n) = n - 1$.

(10) Find $\phi(300)$.

(11) What is the formula for $\phi(p^k)$ and $\phi(pq)$ where p and q are distinct prime.

(12) State Fermat's theorem.

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Candidate's Seat No : _____

B.Sc. Semester-5 Examination

SE 305

Mathematics (B)

March-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions:

1) **All questions are compulsory.**
2) **Notations are usual, everywhere.**
3) **Figures to the right indicate marks of the question/sub-question.**

Q-1 (a) Write a short Note on Time Value of Money. 9

(b) What is the Future value of Rs. 21,000 invested for ten years, for opportunity cost (interest rate) is 5% per year compounded annually, semi-annually, quarterly, monthly, weekly, daily, continuously? 9

OR

(a) Derive the formulas of different compound interest rates and continuous compounded interest rates. 9

(b) What is the Future value of Rs. 40,000 invested for 7 years, for opportunity cost (interest rate) is 35% per year compounded semi-annually, quarterly, monthly, and daily also find effective rate of interest in each case? 9

Q-2 (a) Define yield to maturity for given bond. Show that for a bond of n years with annual coupon payment C and face value F, if its yield (yield to maturity) is λ then its price is given by 9

$$P = \frac{1}{(1+\lambda)^n}\left[\frac{(1+\lambda)^n-1}{\lambda}C + F\right].$$

(b) consider the cash flow with annual payments of -500, 200, -100, 200 suppose the relevant annual compound rates are finance rate is 10% and reinvestment rate 20% find MIRR. 9

OR

(a) Write a short note on comparison of NPV and IRR. 9

(b) A company wants to immunize its bond portfolio for a targeted period of 3 years for this purpose company has decide to invest Rs. 7,00,000/- at present and the details of two bonds are as follows. 9

	Bond A	Bond B
Face Value	1500	2000
Market Price	986.5	1035
Macaulay Duration	5 years	2 years

Determine the amount of money invested in each bond.

Q-3 (a) Write a short note on portfolio diagram and choice of asset. 9

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(b) Calculate the portfolios mean return and variance using the following details, 9

$R = (0.6, 0.7, 0.75)^T$, $W = (0.35, 0.3, 0.35)$ and

$$CV = \begin{bmatrix} 1.2 & 1.4 & 0.9 \\ 1.4 & 2.2 & 0.60 \\ 0.9 & 0.60 & 1.2 \end{bmatrix}$$ find $\bar{r}$ & σ^2 for portfolio.

OR

(a) Discuss Markowitz portfolio optimization problem with short selling and without short selling. 9

(b) Consider a portfolio of three assets, A, B & C with the following properties. 9

$$\overline{r_A} = 0.3 , \overline{r_B} = 0.5 , \overline{r_C} = 0.6,$$
$$\sigma_A = \sigma_B = \sigma_C = 1.5 \ \& \ \sigma_{AB} = \sigma_{BC} = \sigma_{AC} = 0$$

For fixed $\bar{r} = 0.55$ find the minimum variance portfolio.

Q-7 Attempt **any Eight** of the followings **in short:** 16

1. Define inflation and write its formula.
2. Write types of financial instrument.
3. State No Arbitrage Principle.
4. Write a future value of 500 after two years with rate of interest 15% per year compounded annually.
5. Define Puttable Bonds.
6. Write the Formula for Macaulay Duration for discrete compounding.
7. Define Bond.
8. Define quasi-Modified Duration for annual discrete compounding.
9. Define diversification in portfolio.
10. Write the statement of one fund theorem.
11. Define efficient frontier.
12. Define Markowitz bullet.