2709N577 Candidate's Seat No :________

B.Sc Sem.-6 (Rep) Examination

CC 309

Mathematics

Time : 2-30 Hours] **September-2024** **[Max. Marks : 70**

Instruction: All questions are compulsory.

Q-1 (A) Let X be a metric space. Prove that a subset G of X is open if and only if it is a Union of an open spheres [7]

(B) Let X be a metric space. A subset F of X is open if and only if its complement F^c is closed [7]

OR

Q-1 (A) Let (X,d) be a complete metric space and Y be a subspace of X. Then prove that Y is complete if and only if it is closed in (X,d). [7]

(B) Let (X,d) be a metric space. Prove that A is close set if and only if $A=\overline{A}$. [7]

Q-2 (A) Prove that compact subsets of metric spaces are closed. [7]

(B) Let (X,d_1) and (Y,d_2) be metric spaces then prove that $f: X \to Y$ is continuous on X iff $f^{-1}(F)$ is closed in X , whenever F is closed in Y. [7]

OR

Q-2 (A) A subset E of the real line R1 is connected if and only if it has the following property : if $x \in E, y \in E$ *and* $x < z < y$ then $z \in E$

(B) Let X and Y be metric spaces and f a mapping of X into Y then prove that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y. [7]

Q-3 (A) State and prove Weierstrass M_n-test. Show that the sequence $\{f_n(x)\}$ is uniformly convergent on interval [-1,1] where $f_n(x) = \dfrac{x}{1+n^2x^2}$ [7]

(B) Let (f_n) be a sequence of continuous functions on $E \subset C$ converging uniformly to f on E.Then Prove that f is continuous on E. [7]

OR

Q-3 (A) Let f_n satisfy (1) $f_n \in D[a,b]$ (2) $(f_n(x_0))$ converges for $x_0 \in D[a,b]$ (3) f_n' converges uniformly on [a,b] then prove that f_n converges uniformly on [a,b] to a function f. [7]

(B) Let (f_n) be the sequence of functions in R[a,b] converging uniformly to fthen Prove that f is Riemann integrable. Show that $\lim_{n\to\infty} \int_a^b f_n(x)dx = \int_a^b f(x)dx$. [7]

Q-4 (A) State and prove Abel's limit theorem. [7]

(B) For every $x \in R$, and n>0, prove that $\sum_{k=0}^{n} (nx-k)^2 \binom{n}{k} x^k (1-x)^{n-k} = nx(1-x) \le n/4$ [7]

OR

Q-4 (A) State and prove Weierstrass Approximation theorem. [7]

(B) State Taylor's series.

Show that for $-1 \le x \le 1$, $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... + (-1)^{n-1}\frac{x^n}{n} + ...$. Hence evaluate log2.

(P.T.O...)

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Q. 5 Answer in short (any seven) : [14]

1. Define closure of a set.
2. Define metric space.
3. Define Derived set. Give one example of derived set.
4. Define compact set.
5. Define finite subcover.
6. Prove that open interval (0,1) with usual metric is not compact.
7. Is $f_n(x) = \frac{1}{1+nx} (x \geq 0)$ continuous ? justify.
8. If the series $\sum a_k$ converges absolutely then prove that the series $\sum a_k \cos kx$ is uniformly convergent on R.
9. Show that the series $\sum_{k=0}^{\infty} (xe^{-x})^k$ is uniformly convergent.
10. Prove by Taylor's series $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$
11. State Bernstein Theorem.
12. For every $x \in R$ and $n \geq 0$ prove that $\sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} = 1$

----- ---------*Best of Luck*-----------