2703E1361

Candidate's Seat No :________

B.Sc. Semester-5 Examination
CC 304
Mathematics
March-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instruction : (i) All the questions are compulsory and carry equal marks.
(ii) Notations are usual everywhere.
(iii) The right hand side figures indicate marks of the question/sub-question.

Q 1 (*a*) Prove that $K \subset R^n$ is a convex set if and only if every convex linear combination of elements in K belongs to K. 7

(*b*) If $S_1 = \{(x,y) \in R^2 / x^2 + y^2 \le 9\}$ and $S_2 = \{(x,y) \in R^2 / x^2 + y^2 \ge 9\}$. Then determine the convexity of the sets $S_1 and\ S_2$. Also obtain $S_1 \cup S_2$. Is $S_1 \cup S_2$ convex? Justify your answer. 7

OR

Q 1 (*a*) Prove that the nonempty set S_F of all feasible solutions of a LP Problem is a convex set. 7

(*b*) A Firm manufactures two products: Chairs and Tables. These products are processed on two machines A and B. A chair requires 2 hours of processing time on machine A and 3 hours on machine B. A table requires 5 hours of processing time on machine A and no time on machine B. There are 16 hours of time available for machine A and 20 hours on machine B during any working day. Profit gained by the firm per a chair is Rs 40 and that per a table is Rs 60. 7

What should be the daily production of each of the two products to maximize total profit of the firm?

Formulate the linear programming problem.

Q 2 (*a*) Explain Simplex algorithm for solving linear programming problem. 7

(*b*) Solve the following LPP by Two Phase Method: 7

$$\text{Maximize } Z = 3x_1 + 2x_2$$
$$\text{Subject to } 2x_1 + x_2 \le 2$$
$$3x_1 + 4x_2 \ge 12 \quad \text{and} \quad x_1, x_2 \ge 0.$$

OR

P.T.O

Q 2 (*a*) Apply the Simplex Method to Maximize $Z = 10x_1+x_2+2x_3$ 7

Subject to $x_1+ x_2 - 2x_3 \leq 10$

$4x_1+x_2+ x_3 \leq 20$ and $x_1, x_2, x_3 \geq 0$.

(*b*) Solve the following LPP by big-M Method : 7

Maximize $Z = 3x_1 + 2x_2 + 3x_3$

Subject to $2x_1+x_2 + x_3 \leq 2$

$3x_1 + 4x_2+2x_3 \geq 8$ and $x_1, x_2, x_3 \geq 0$.

Q 3 (*a*) Prove that a Dual of a Dual is the primal LP Problem. 7

(*b*) Use the Dual Simplex Method to solve the following LP Problem: 7

Minimize $Z = 2x_1+ x_2 + x_3$

Subject to $4x_1+6x_2+3x_3 \leq 8$

$-x_1+9x_2 - x_3 \geq 3$

$-2x_1-3x_2+5x_3 \leq -4$ and $x_1, x_2, x_3 \geq 0$.

OR

Q 3 (*a*) Explain Duality and advantages of duality. 7

(*b*) Solve the following integer linear programming problem by cutting plane method : 7

Maximize $Z = x_1 + x_2$

Subject to $3x_1 + 2x_2 \leq 5$

$x_1 \leq 2$

$x_1, x_2 \geq 0$ and are integers.

Q 4 (*a*) What is a transportation problem.? 7

Explain how is it a special case of the Linear Programming problem.

(*b*) Solve the following Transportation Problem by MODI Method : 7

Factory	Warehouses				Capacity
	W_1	W_2	W_3	W_4	
F_1	15	26	46	6	7
F_2	66	26	36	56	9
F_3	36	4	66	16	18
Requirements	5	8	7	14	34

OR

Q 4 (*a*) Prove that the necessary and sufficient condition for the existence of feasible solution to the TP is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. 7

(*b*) Solve the following Assignment Problem: 7

	A	B	C	D
I	38	33	36	39
II	40	33	32	38
III	36	32	32	35
IV	30	28	26	32

Q5 Answer any **SEVEN** of the followings in Short : **14**

(*a*) Define: Vertex of a convex set

(*b*) Define: A Convex Polyhedron.

(*c*) Define: An Artificial Variable.

(*d*) Define: Standard form of a LP Problem.

(*e*) When a LP Problem is said to have an unbounded solution?

(*f*) When the Dual Simplex Method is applicable to solve a LP Problem ?

(*g*) Find the Standard primal form of the following LPP:
$Maximize\ Z = 4x_1 + 3x_2 + 2x_3$
Subject to constraints: $2x_1 - x_2 + x_3 \le 3$
$x_1 + x_2 - x_3 \ge 4$
$x_1 + x_2 + x_3 = 5$ and $x_1, x_2, x_3 \ge 0$

Determine True or False for the following statements :

(*h*) Every integer linear programming problem is a linear Programming Problem.

(*i*) If Dual LPP has an unbounded solution then primal has no feasible solution.

(*j*) Every Linear Programming Problem has a solution.

(*k*) Every transportation problem is an assignment Problem.

(*l*) An assignment Problem is solved by the Hungarian Method.

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