Seat No. : _______________

N12-121

November-2014

M.Sc. (CBCS) Sem.-III

STA-501 : Statistics

(Testing of Hypothesis)

Time : 3 Hours]

Instructions : (1) **All** questions are of equal marks.
(2) Scientific calculator is permitted to use.
(3) Statistical Tables will be supplied on request.

1. (a) Define randomized test function. State and prove sufficient part of NP lemma for randomized test.

OR

Prove or disprove : The test obtained by NP lemma is essentially unbiased.

(b) Let X and Y are two independent normal variates with mean θ and 2θ respectively with common variance 1. Derive most powerful test of size α to test $H : \theta = \theta_0$ versus $K : \theta = \theta_1, \theta_1 > \theta_0$. Obtain also power of the test when $\theta_1 = 3$, $\theta_0 = 2$ and $\alpha = 0.05$.

OR

Let X be a random variable having the pmf f or g. Derive most powerful test of size α to test

$H : X \sim f = \frac{1}{2^{x+1}}$, $x = 0, 1, 2, \ldots$ versus $K : X \sim g = \frac{1}{4}\left(\frac{3}{4}\right)^x$, $x = 0, 1, 2, \ldots$

Also obtain power of the test.

2. (a) Define MLR property of a distribution. State an exponential family of distributions. Obtain sufficient condition for the distribution to possess an MLR property. Verify it for the distribution having pdf $f(x,\theta) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta > 0$.

OR

If the pdf $f(x, \theta)$ has MLR property in $T(x)$; show that there exist a UMP test for testing $H : \theta \leq \theta_0$ versus $K : \theta > \theta_0$ based on $T(x)$.

(b) Obtain UMP test for testing H : $\theta = \theta_0$ versus K : $\theta \neq \theta_0$ based on random sample of size n taken from uniform U(0, θ), $\theta > 0$ distribution. Hence derive $(1 - \alpha)100\%$ UMA confidence interval for θ.

OR

Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with pdf $f(x, \theta) = \theta x^{\theta - 1}$, $0 < x < 1$, $\theta > 0$. Obtain UMPU test for testing H : $\theta = \theta_0$ versus K : $\theta \neq \theta_0$. Hence derive $(1 - \alpha)100\%$ UMAU confidence interval for θ.

3. (a) Discuss the test procedure for testing the hypothesis in the presence of nuisance parameter(s) with example.

OR

Let $X_1, X_2, \ldots, X_n$ be a random sample from N(μ, σ^2), distribution. To test H : $\mu = \mu_0$ versus K : $\mu \neq \mu_0$ derive LRT of size α.

(b) Describe SPRT procedure. Obtain relation between stopping bounds and strengths of an SPRT.

OR

Prove that SPRT eventually terminates with probability one.

4. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from the uniform U(0, θ), $\theta > 0$ distribution. Derive SPRT to test H : $\theta = \theta_0$ versus K : $\theta = \theta_1$, $\theta_1 > \theta_0$.

OR

Let $X_1, X_2, \ldots, X_n$ be a random sample from N(μ, 4) distribution. Obtain SPRT to test H : $\mu = -1$ versus K : $\mu = 1$. Find also E(N) under H and K.

(b) Describe fully Kolmogorov – Smirnov test.

OR

Describe fully Kruskal – Wallis test.

5. Answer the following :

(i) Define size and power of the randomized test.

(ii) Define UMP test.

(iii) State the necessary condition for the existence of UMP test with two sided alternative.

(iv) Define UMPU test.

(v) Define boundary set.

(vi) Define similar region test.

(vii) State asymptotic distribution of $-2\log\lambda(x)$ in LRT for testing $H : \theta = \theta_0$ versus $K : \theta = \theta_1$.

(viii) The pdf $f(x, \theta) = \theta/x^2$, $0 < \theta < x < \infty$

(A) possess MLR property in $T(x) = X_{(1)}$

(B) possess MLR property in $T(x) = X_{(n)}$

(C) possess MLR property in $T(x) = \sum_{i=1}^{n} X_i^2$

(D) does not possess MLR property

(ix) Let $X \sim N(0,1)$ under H and N(0, 2) under K. Suppose the $\varphi(x) = 1$, if $|x| \geq 1$ and zero otherwise, then find size and power of the test.

(x) Say TRUE or FALSE: If UMP test exist LRT always provide it.

(xi) State Wald's identity of an SPRT.

(xii) Find the value of the test statistic involved in the Kolmogorov – Smirnov test to test whether the random sample {2.5, 3.9, 0.8} is taken from U(0, 4) uniform distribution or not.