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2609N544

Candidate's Seat No :________

B.Sc Sem.-6 (Rep) Examination
CC 308
Mathematics
September-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: (1) All questions are compulsory.
(2) Figure to the right indicates full marks of the question.

Q-1 (a) Let f be integrable on [a,b] and $a < c < b$ then prove that f is integrable on [a,c] and [c,b] and $\int_a^b f = \int_a^c f + \int_c^b f$ **(7)**

(b) Let $f(x)=x/4$ on [0,1] for $n \in N$, $P_n=\left\{0,\frac{1}{n},\frac{2}{n},\frac{3}{n},\frac{4}{n}\ldots\frac{n-1}{n},1\right\}$ then find $\lim_{n\to\infty} U[f;P_n]$ *and* $\lim_{n\to\infty} L[f;P_n]$. **(7)**

OR

Q-1 (a) Let f be bounded on [a,b]. Then prove that f is integrable on [a,b] if and only if for every $\varepsilon > 0$, there is partition P for which $U_p(f) - L_p(f) < \varepsilon$ **(7)**

(b) Test for convergence: **(7)**

(i) $\int_0^1 \frac{dx}{x^3(1+x)}$ (ii) $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}dx$

Q-2 (a) State and prove Cauchy's Condensation Test. **(7)**

(b) Test for convergence: **(7)**

(i) $\sum_{n=1}^{\infty} \frac{n^{1/2}}{n^2+5n+1}$ (ii) $\sum_{n=1}^{\infty} \left(1+\frac{3}{n}\right)^{-n^2}$

OR

Q-2 (a) Let (x_n) be a bounded sequence of real numbers. Prove that (x_n) is a convergent sequence of real numbers if and only if **(7)**

$\underline{\lim}\, x_n = \overline{\lim}\, x_n = \lim_{n\to\infty} x_n$.

(b) Prove that if p > 1, the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges and if $p \le 1$, the series diverges. **(7)**

Q-3 (a) State and prove Mertens' theorem. **(7)**

(b) For the following, determine whether the series converges absolutely, converges conditionally, or diverges: **(7)**

(i) $\sum_{n=1}^{\infty} \frac{(-1)^n n^{1/2}}{(n+3)}$ (ii) $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^3}$.

OR

Q-3 (a) State and prove Leibnitz Alternating Series test. **(7)**

(b) Find the set of convergence (interval of convergence) and radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{n^3(x-3)^n}{(n^2+2)3^n}$. **(7)**

[P.T.O.]

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Q-4 (a) Let f be a real valued function on $[a,a+h]$ and $f^{n+1}(x)$ is continuous on $[a,a+h]$. Then prove that **(7)**

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_{n+1}(x)$$

for $x \in [a, a+h]$

Where $R_{n+1}(x) = \frac{1}{n!}\int_a^x (x-t)^n f^{(n+1)}(t)dt.$

(b) Write down Taylor's formula with Lagrange form of remainder for $f(x) = \log(1+x)$ about $a=2$ and $n=4$. **(7)**

OR

Q-4 (a) State and prove binomial theorem. **(7)**

(b) Find the power series solution of $(1-x)y'-2y=0$ with the condition $y(0)=4$. **(7)**

Q-5 **Attempt any Seven short questions:** **(14)**

(i) Find the primitive F of $f(x) = e^{2x} - 5\sin x$

(ii) Find whether $f(x)=1/(x+3)$ is Riemann-integrable on [0,2] and justify your answer.

(iii) Evaluate: $\int_0^6 [x]dx$

(iv) Find limit superior and limit inferior of the sequence: $s_n = \{1,-1,1,-2,1,-3,1,-4,..\}$

(v) Give an example of series for which both ratio and root tests fail.

(vi) Find $\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n$,

(vii) Give an example of power series with 0 radius of convergence.

(viii) Define: Absolute Convergence

(ix) Define Cauchy product of series.

(x) State the series of $\cos x$, for any real x.

(xi) Find power series solution of $y'=y$.

(xii) State Taylor's formula with Cauchy's form of remainder.

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