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2609N532

Candidate's Seat No :________

M.Sc Sem.-4 (Rep) Examination

509

Mathematics (EA)

September-2024

Time : 2-30 Hours] **[Max. Marks : 70**

1. (A) Find all solutions in the positive integers of $172x + 20y = 1000$. **7**

   (B) Find the formula for $\sigma(n)$, for $n > 1$. Calculate $\sigma(2024)$. **7**

OR

1. (A) If n is a positive integer and p a prime, prove that the exponent of the highest power of p that divides $n!$ is $\sum_{k=1}^{\infty} \left[\frac{n}{p^k}\right]$. **7**

   (B) Prove that the functions τ, σ and μ are multiplicative functions. **7**

2. (A) State Chinese remainder theorem.(Do not Prove.)
   Solve: $x \equiv 3 \pmod 6$, $x \equiv 5 \pmod 7$, $x \equiv 2 \pmod{11}$. **7**

   (B) State and prove Fermat's theorem.
   Find the unit digit of 3^{100} by the use of Fermat's theorem. **7**

OR

2. (A) State and prove Wilson's theorem. **7**

   (B) Construct a table of indices for the prime 13 with respect to the primitive root 2. Using this table, solve $4x^9 \equiv 7 \pmod{13}$. **7**

3. (A) Solve the quadratic congruence $3x^2 + 9x + 7 \equiv 0 \pmod{13}$. **7**

   (B) Prove that there are infinitely many primes of the form $8k - 1$. **7**

OR

3. (A) State and prove Euler's criterion. **7**

   (B) Solve the congruence $x^2 \equiv 7 \pmod{3^3}$. **7**

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4. (A) Express $\frac{187}{57}$ and $\frac{118}{303}$ as finite simple continued fractions. **7**

(B) Determine the general solution of $364x + 227y = 1$ by means of simple continued fractions. **7**

OR

4. (A) Determine the infinite continued fraction representation of $\sqrt{23}$. **7**

(B) Evaluate $[3; 6, \overline{1, 4}]$. **7**

5. **Attempt any seven of the following.** **14**

(1) The number of positive divisors of 5000 is

(A) 22 (B) 24 (C) 12 (D) 20

(2) $\sum_{d|3500} \mu(d) =$ _____, where d runs through the positive divisors of 3500.

(A) 1 (B) -1 (C) 0 (D) 2

(3) If $a^n - 1$ is prime $(a > 0, n \geq 2)$, then

(A) $a = 2$ and n is prime.

(B) $a = 2$ and n is composite.

(C) $a = 3$ and n is prime.

(D) $a = 3$ and n is composite.

(4) The sum of the positive integers less than 100 and relatively prime to 100 is

(A) 40 (B) 2000 (C) 1000 (D) 4000

(5) Which of the following congruences are solvable?

(A) $27x \equiv 13 \pmod{31}$

(B) $22x \equiv 28 \pmod{44}$

(C) $17x \equiv 54 \pmod{102}$

(D) $19x \equiv 78 \pmod{91}$

(6) The number of the incongruent primitive roots of 31 is

(A) 4 (B) 8 (C) 30 (D) 31

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(7) What is the remainder when 2^{50} is divided by 7?

(A) 2 (B) 4 (C) 5 (D) 0

(8) Which of the following are quadratic residues of 13?

(A) 1 (B) 3 (C) 8 (D) 12

(9) For which of the following prime numbers, $(p/q) = (q/p)$?

(A) $p = 11$, $q = 7$
(B) $p = 13$, $q = 5$
(C) $p = 19$, $q = 11$
(D) $p = 23$, $q = 17$

(10) Which of the following Legendre symbols has the value 1?

(A) (2/13) (B) (5/19) (C) (18/43) (D) (19/23)

(11) The third convergent C_3 of the continued faction $[0; 2, 1, 2, 6]$ has the value

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$

(12) The rational number represented by $[4; 2, 1, 3, 1, 2, 4]$ is

(A) $\frac{741}{170}$ (B) $\frac{170}{741}$ (C) $\frac{680}{170}$ (D) $\frac{741}{61}$

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Mathematics (EB)

September-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Find the Laplace transform of the function $\frac{1}{t^2}(1-\cos t)$ **(07)**

(b) State the convolution theorem and find the inverse Laplace transform of $\frac{1}{(s^2+1)^3}$ **(07)**

OR

Q.1 **(a)** Using the Laplace transforms, find the solution of the initial value problem $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=0$, where $y=2, \frac{dy}{dx}=-4$ at $x=0$ **(07)**

(b) Find the inverse Laplace transform of $\frac{1}{(s-2)(s^2+1)}$ by partial fraction method. **(07)**

Q.2 **(a)** Find the eigenvalues and eigenfunctions of the Strum-Liouville problem: **(07)**
$y''+\mu y=0, \quad y(0)=0, \quad y(L)=0$

(b) Define odd and even function. **(07)**
Are the following functions even or odd or neither even nor odd?
I. e^x
II. $\sin^2 x$
III. Product of an odd times an even function

OR

Q.2 **(a)** Find the Fourier series of $f(x)=x^2$ in the interval $(0,2\pi)$. **(07)**

(b) Find the Fourier cosine and sine transform of $f(x)=e^{-ax}$, where $a>0$. **(07)**

Q.3 **(a)** Find Z-transform of $\cosh\left(\frac{k\pi}{2}+\alpha\right), k>0$ **(07)**

(b) Find Z-transform of $\sin(5k+3), k\geq 0$ **(07)**

OR

Q.3 **(a)** Solve the difference equation $6y_{k+2}-y_{k+1}-y_k=0, y(0)=0, y(1)=1$ by Z-transform. **(07)**

(b) Find inverse Z-transform of $\frac{z}{(z-1)(z-2)}$ by residue method. **(07)**

Q.4 **(a)** Show that $\int_0^a xJ_0(sx)dx=J_1(as)\cdot\left(\frac{a}{s}\right)$ **(07)**

(b) Show that if n = 0, the Hankel transform **(07)**

$$H\left\{\frac{\cos ax}{x}\right\}=\begin{cases}0 & if\ s<a\\ \frac{1}{\sqrt{s^2-a^2}} & if\ s>a\end{cases}$$

OR

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Q.4 **(a)** Find the Hankel transform of **(07)**

$$f(x) = \begin{cases} (a^2 - x^2) & 0 < x < a, \quad n = 0 \\ 0 & x > a, \quad n = 0 \end{cases}$$

(b) Prove Linearity property of Hankel transform and find the Hankel transform of **(07)**

$$f(x) = \begin{cases} 1 & 0 < x < a, \quad n = 0 \\ 0 & x > a, \quad n = 0 \end{cases}$$

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Find Laplace transform of $t^3 e^{-2t}$

(2) Find the inverse Laplace transform of $\frac{e^{-3s}}{s^3}$

(3) Find Laplace transform of $sint\ u(t-4)$

(4) Find the fundamental period of $\cos 2x, \sin 2\pi x$

(5) Find the constant a_0 of the Fourier series for the function $f(x) = e^x$ in the interval $-\pi < x < \pi$.

(6) State Parseval's identity.

(7) Write down the sequence $\frac{1}{2}\{f(k)\}$ where $f(k) = \frac{1}{3^k}$

(8) Find inverse Z-transform of $\frac{1}{z-2}, |z| > 2$

(9) Find Z transform of the sequence {8, 6, 3, -1, 0, 14, 5}

(10) Define: Hankel transform

(11) Find $H^{-1}[e^{-as}]$ when $n = 1$

(12) Find the Hankel transform of $e^{-ax}, n = 0$

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