

## Integ. MSc AIML Semester-2 Examination

CC 112

## Numerical &amp; Statistical Methods

Time : 2-30 Hours]

April-2024

[Max. Marks : 70]

**Instructions:** All questions are compulsory. Use of non-programmable scientific calculator is allowed.

**Q.1** (a) Perform the five iterations of the bisection method to obtain a root of the equation  $f(x) := x^3 - 9x + 1 = 0$  (6/7)

(b) Apply Budan's theorem to find the number of roots of the equation  $x^5 - x^4 - 4x^3 + 9x^2 - x + 5$  in the intervals  $[-3, -2]$  and  $[-2, -1]$ . (6/7)

**OR**

(a) Derive an iterative formula to find  $\sqrt{N}$  and hence find approximate value of  $\sqrt{15}$  correct up to three decimal places. (6/7)

(b) Use Newton-Raphson method to find the positive root of  $f(x) = x^3 - 5x + 1 = 0$  correct to three decimals. (6/7)

**Q.2** (a) Estimate the production for the year 1985 with the help of the following table. (6/7)

| Year | Production (in tonnes) |
|------|------------------------|
| 1960 | 20                     |
| 1965 | 22                     |
| 1970 | 26                     |
| 1975 | 30                     |
| 1980 | 35                     |
| 1985 | ?                      |
| 1990 | 43                     |

(b) Determine the percentage of criminals under 35 years of age. (Using Lagrange's method) (6/7)

| Age            | Percentage of criminals |
|----------------|-------------------------|
| Under 25 years | 52.0                    |
| Under 25 years | 67.3                    |
| Under 25 years | 84.1                    |
| Under 25 years | 94.4                    |

**OR**

(P.T.O)

(a) Extrapolate the business done in 1999 from the following data: (07)

| Year | Business done (Rs. Lakhs) |
|------|---------------------------|
| 1994 | 150                       |
| 1995 | 235                       |
| 1996 | 365                       |
| 1997 | 525                       |
| 1998 | 780                       |

(b) Below are given the wages earned by workers per week in a certain factory. Calculate the number of workers earning more than Rs. 750 per week. (07)

| Weekly wages   | No. of workers |
|----------------|----------------|
| Up to Rs. 500  | 50             |
| Up to Rs. 600  | 150            |
| Up to Rs. 700  | 300            |
| Up to Rs. 800  | 500            |
| Up to Rs. 900  | 700            |
| Up to Rs. 1000 | 800            |

Q.3 (a) Explain in brief: The Method of Least squares. (07)  
 (b) Fit the transcendental equation  $y = ax^b$  to the data given below. (07)

|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| $x$ | 2   | 4   | 6   | 8   |
| $y$ | 1.4 | 2.0 | 2.4 | 2.6 |

OR

(a) Fit a parabola of the form  $y = ax^2 + bx + c$  to the following data: (07)

|     |   |     |     |     |     |
|-----|---|-----|-----|-----|-----|
| $x$ | 0 | 1   | 2   | 3   | 4   |
| $y$ | 1 | 1.8 | 1.3 | 2.5 | 6.3 |

(b) Fit the exponential curve  $y = ae^{bx}$  to the data given below. (07)

|     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|
| $x$ | 0.4 | 0.8 | 1.2 | 1.6 | 2.0 | 2.4 |
| $y$ | 75  | 100 | 140 | 200 | 270 | 375 |

Q.4 (a) A department of transportation's study on driving speed and miles per gallon for midsize automobiles resulted in the following data: (07)

|                  |    |    |    |    |    |    |    |    |    |    |
|------------------|----|----|----|----|----|----|----|----|----|----|
| SPEED            | 30 | 50 | 40 | 55 | 30 | 25 | 60 | 25 | 50 | 55 |
| Miles Per Gallon | 28 | 25 | 25 | 23 | 30 | 32 | 21 | 35 | 26 | 25 |

Compute and interpret the sample correlation coefficient.

(b) Given are five observations for two variables  $x$  and  $y$ .

(07)

|     |    |    |    |    |    |
|-----|----|----|----|----|----|
| $x$ | 3  | 12 | 6  | 20 | 14 |
| $y$ | 55 | 40 | 55 | 10 | 15 |

(a) Develop a scatter diagram for these data.

(b) Develop the estimated regression equation by computing the values of  $b_0$  and  $b_1$  using equations.

Use regression equation to predict the value of  $y$  when  $x = 10$ .

**OR**

(a) Discuss all types of linear regression with giving at least one example.

(07)

(b) The daily high and low temperatures for 14 cities around the world are shown:

| City       | High | Low |
|------------|------|-----|
| Ahmedabad  | 68   | 50  |
| Baroda     | 70   | 49  |
| Junagadh   | 65   | 44  |
| Bhavnagar  | 96   | 64  |
| Surat      | 57   | 46  |
| Rajkot     | 70   | 45  |
| Kutch      | 80   | 73  |
| Palanpur   | 67   | 45  |
| Mehsana    | 44   | 29  |
| Dang       | 69   | 44  |
| Panchmahal | 76   | 69  |
| Patan      | 69   | 51  |
| Kheda      | 70   | 58  |
| Nadiad     | 44   | 39  |

(a) What is the sample mean high temperature?

(b) What is the sample mean low temperature?

(c) What is the correlation between the high and low temperatures? Discuss.

**Q.5** Attempt any **SEVEN** out of **TWELVE**:

(14)

- (1) State Descartes' rule of signs with suitable example.
- (2) Find the number of the positive roots of the polynomial  $x^3 + 3x^2 - x - x^4 - 2$
- (3) Give normal equations of a Geometric curve  $y = ab^x$
- (4) Give normal equations of the exponential curve  $y = ae^{bx}$
- (5) Define: Interpolation and Extrapolation with suitable example.
- (6) Prepare a table of divided differences of different orders.

|                       |      |      |      |      |
|-----------------------|------|------|------|------|
| Year (x)              | 2011 | 2015 | 2017 | 2020 |
| Amount of Pension (y) | 105  | 191  | 309  | 487  |

- (7) What is Correlation? State its types.
- (8) Define: Relative and Percentage error with suitable example.
- (9) Write the formula of Lagrange's interpolation method.
- (10) For regression line  $Y = 3.76 + 0.4 X$ , what is the estimated value of  $Y$  for  $X = 12$ ?
- (11) If two variables are having ranks in reverse order, write the value of correlation coefficient.
- (12) What is the order of convergence of Newton-Raphson's iterative method.

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